STAT 2507 Solution-Assignment # 2 Fall 2016

Total mark 100

Part I. Lab questions.

• Do not include ANY Minitab code to your assignment. Write your answers for the Minitab questions on the provided lines.

1. (Mean and variance of random variables)
   Consider the random variable X whose pmf is

x	1	2	3	4	5	6	7	8
$P(X=x)$	0.004	0.021	0.074	0.171	0.267	0.267	0.156	0.04

Use the following Minitab commands to enter the pmf of X into Minitab so that the possible values of X are stored in column c1 and the probabilities are stored in column c2 by:
(First click **Editor** → **Enable commands**) and type the below commands
MTB> set c1
DATA> 1 2 3 4 5 6 7 8
DATA> end
MTB> set c2
DATA> 0.004 0.021 0.074 0.171 0.267 0.267 0.156 0.04
DATA> end

a. Obtain the mean of X, i.e. $\mu = E(X)$ by typing the following command
MTB>let c8= sum(c1*c2)
(the mean will appear in the first cell of column c8)

What is μ? [2] —— **5.301** ———.

b. By typing the following command obtain the variance $Var(X) = \sigma^2$ of X.
MTB>let c9= sum((c1**2)*c2) - (sum(c1*c2))**2
(the variance will appear in the first cell of column c9)

What is σ^2 ? [2]—— **1.08804** ————-.

c. What is the the standard deviation σ? [2] ——- **approx. 1.371** ————.

2. (Binomial Distribution)
   Suppose that X has a binomial distribution with n = 40 and p = 0.1. Use Minitab to simulate 50 values of X.

   MTB>random 50 c1; (don't forget to enter the semicolon at the end)
   SUBC>binomial 40 0.1. (don't forget to enter the dot at the end)

   a. What proportion of your values are strictly less than 5? [2] —— **I had 34/50=0.68** ————-.
   Hint: to answer part (a) you can sort the simulated values of column c1 in another column, say c7 by typing the command:
   MTB> let c7=sort(c1)

   b. What is the exact probability that X will be strictly less than 5? [2] —- **0.62902** ————.
   c. Using this cdf also obtain the following probabilities
   $P(X \leq 6)$ [2] —— **0.90048** ————-
   $P(X = 8)$ [2] ——- **0.026405** ————-

   Hint: To answer the preceding parts (b) and (c) you need to find $P(X \leq k)$ for any $k = 0, \ldots, 40$, use "cdf" command; this works by typing
   MTB>cdf;
   SUBC>binomial 40 0.1.

   d. Find the value a such that $P(X \leq a) = 0.98450$. [2] —— **8** — ———
   To answer this question you need to use the "inverse cdf" command by typing
   MTB>invcdf 0.98450;
   SUBC>binomial 40 0.1.

3. (Poisson Distribution)
   Suppose that Y has a Poisson distribution with mean $\mu = 4$. Use the cdf command
   MTB>cdf;
   SUBC>poisson 4.

   Find [2] $P(Y \leq 6) =$ —— **0.88933** — and [2] $P(Y = 8) =$ — **0.02977** —— .

4. (Poisson approximation to Binomial) Refer the Questions 2 and 3.
   a. Compare $P(X \leq 6)$ you obtained in part (**b**) of Question 2 for the

binomial random variable X to $P(Y \leq 6)$ you obtained in Question 3 for Poisson with mean $\mu = 4$.
Are their values relatively close to each other? [1] —- **Yes they are fairly close** ——.
b. Compare $P(X = 8))$ you obtained in part (**b**) of Question 2 for the binomial random variable X to $P(Y = 8)$ you obtained in Question 3 for Poisson with mean $\mu = 4$.
Are their values relatively close to each other? [1] —— **Yes they are fairly close** —–.
c. Should these probabilities in each of part (a) and part (b) of this question be close to each other or not? [1] — **Yes** —

d. Explain why they should be close or why they should not be close? [3] —- **Since for the binomial X we have $np = 4 < 7$, the poisson approximation to binomial was expected to be a good one** —–

5. (Hypergeometric Distribution)
In a party there is an ice bin that contains 20 cans of soft drink of which 8 are Pepsi and the rest are Cokes. All cans are covered by ice so we cannot see their brand. If we randomly select 5 cans without replacement from the ice bin and let H be the number of Pepsi among the the selected 5. We know that H has hypergeometric distribution with $N = 20$, $M = 8$ and $n = 5$.

Derive the cdf of H by:
Type 0, 1, 2, 3, 4, 5 in column c10, use the command
MTB>cdf c10;
SUBC>hypergeometric 20 8 5.

Using the cdf of H answer the following parts(a) and (b).

a. What is the probability that there will be at most 2 (inclusive) Pepsi among the chosen drinks. [2] —— **0.70382** ——-

b. What is the probability that there will exactly 2 cokes among the chosen drinks. [2] ——— **0.23839** ———

Part II. Long-answer questions; Give the solutions for the following questions in details

1. An aerospace company has submitted bids on two separate federal government defense contracts, A and B. The company feels that it has a 60% chance of winning contract A and a 30% chance of winning contract B. If it wins contract B, it believes that it has an 80% chance of winning contract A.

   a) [2] Are the events of winning contract A and winning contract B independent? Explain.
   b) [2] What is the probability that the company will win both contracts?
   c) [2] Are the events of winning contract A and winning contract B mutually exclusive? Explain.
   d) [2] What is the probability that the company will win at least one of the contracts?
   e) [2] What is the probability that the company will win neither contract?
   f) [2] What is the probability that the company will win contract A but not contract B?
   g) [2] If the company wins contract A, what is the probability that it will win contract B?
   h) [2] If the company wins contract B, what is the probability that it will not win contract A ?

   Solution:
   $P(A) = 0.60,\ P(B) = 0.30,\ P(A|B) = 0.80$
   a) A and B are not independent. Since,

   $$P(A|B) = 0.8 \neq P(A) = 0.6$$

   b) P(both contracts) = $P(A \cap B) = P(A|B)P(B) = 0.8(0.3) = 0.24$
   c) A and B are not mutually exclusive. Since, if they were mutually exclusive, then $P(A \cap B) = 0$. But $P(A \cap B) = 0.24$
   d) P(at least 1 contract) $= P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.60 + 0.30 - 0.24 = 0.66$
   e) P(neither contract) = 1-P(at least 1 contract) $= 1 - P(A \cup B) = 1 - 0.66 = 0.34$
   f) P(A only) $= P(A \cap B') = P(A) - P(A \cap B) = 0.6 - 0.24 = 0.36$
   g) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.24}{0.6} = 0.40$
   h) $P(A'|B) = \frac{P(A' \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)} = \frac{0.3 - 0.24}{0.3} = \frac{0.06}{0.24} = 0.2$

2. Let $P(A|B) = 0.5,\ P(B) = 0.25,\ P(A \cup B) = 0.75$.
   a) [6] Find (i) $P(A \cap B)$, (ii) $P(A)$, (iii) $P(B'|A')$
   (Hint: $(A' \cap B') = (A \cup B)'$ "De morgan Law")

   b) [1] Are A and B independent events?

   Solution:
   a) (i) $P(A \cap B) = P(A|B)P(B) = 0.5(0.25) = 0.125$
   (ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B) \rightarrow P(A) = P(A \cup B) - P(B) + P(A \cap B) = 0.75 - 0.25 + 0.125 = 0.625$
   (iii) $P(B'|A') = \frac{P(A' \cap B')}{P(A')} = \frac{P(A \cup B)'}{P(A')} = \frac{1-P(A \cup B)}{1-P(A)} = \frac{1-0.75}{1-0.625} = 0.6666$

   b) A and B are not independent events, since $P(A|B) = 0.5 \neq 0.625 = P(A)$

3. The results of three flips of a biased coin are observed. The probability of a head occurring is 0.8. Consider the following events:

   A: At least two tails are observed B: exactly one head is observed C: exactly two heads are observed

   a) [2] List an appropriate sample space S for this experiment.
   b) [1] Are the events of getting a head on flip independent for the 3 flips?
   c) [3] Assign probabilities to the outcomes in S.
   d) [3] Find (i) $P(A)$ (ii) $P(A \cup B)$ (iii) $P(B|A)$
   e) [2] If the random variable X= number of heads in 3 flips, set up the probability distribution in table form.

   Solution:
   3 coins, $P(H) = 0.80,\ P(T) = 0.20$
   a) S={HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}
   b) Yes
   c) $P(HHH) = (0.8)(0.8)(0.8) = (0.8)^3 = 0.512$
   $P(HHT) = P(HTH) = P(THH) = (0.8)^2(0.2) = 0.128$
   $P(HTT) = P(THT) = P(TTH) = (0.8)(0.2)^2 = 0.032$
   $P(TTT) = (0.2)(0.2)(0.2) = (0.2)^3 = 0.008$
   d) (i) $P(A) = P(HTT) + P(THT) + P(TTH) + P(TTT) = 0.032 + 0.032 + 0.032 + 0.008 == 0.104$
   (ii) $P(A \cup B) = P(A) = 0.104$
   (iii) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{P(B)}{P(A)} = \frac{0.096}{0.104} = 0.23$

d) X= # of heads in 3 coin flips , x={ 0, 1, 2, 3}

x	0	1	2	3
$p(x)$	0.008	0.096	0.384	0.512

4. Each week a retail outlet accepts delivery of a certain item from 3 different suppliers, A, B and C. All the items received are put into an empty bin. A supplies 50% of these items, while B and C each supply 25%. From past experience, it is known that 2% of the items supplied by A are defective, 2% of the items supplied by B are defective and 4% of the items supplied by C are defective. Suppose an item is chosen at random from the bin.

   a)[3] What is the probability that it is defective?
   b) [3] What is the probability that it came from supplier C?

   Solution:
   A: event that item comes from supplier A, P(A) = 0.50
   B : event that item comes from supplier B, P(B) = 0.25
   C : event that item comes from supplier C, P(C) = 0.25
   Let D be an event that item is defective

$$P(D|A) = 0.02, P(D|B) = 0.02, P(D|C) = 0.04$$

   a)$P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D) = P(D|A)P(A) + P(D|B)P(B) + P(D|C)P(C) == 0.50(0.02) + 0.25(0.02) + 0.25(0.04) = 0.01 + 0.005 + 0.01 = 0.025$

   b)$P(C|D) = \frac{P(C \cap D)}{P(D)} = \frac{P(D|C)P(C)}{P(D)} = \frac{0.04(0.25)}{0.025} = 0.40$

5. When circuit boards used in the manufacture of compact disc players are tested, the long run percentage of defective is 5%. Let X= the number of defective boards in random sample of size $n = 25$, so $X\ Bin(25, 0.05)$.
   a) [6] Find the probability that the number of defective boards is
   (i) At most 2 (ii) At least 5 (iii) Between 1 and 4, inclusive.
   b) [1] What is the probability that none of the 25 boards is defective?
   c) [2] Calculate the expected value and standard deviation of X.

   Solution:
   $X\ Bin(25, .05)$

a) (i) P(at most 2) $=P(X \leq 2) = P(X=0) + P(X=1) + P(X=2) = 0.873$

(ii) P(at least 5) $=P(X \geq 5) = 9 - P(X \leq 4) = 1 - .007 = 0.993$
(iii) $P(1 \leq X \leq 4) = P(X \leq 4) - P(X \leq 0) = 0.993 - 0.0.278 = 0.715$
b) $P(X=0) = 0.2774$
c) $E(X) = np = 25(0.05) = 1.25$,
$V(X) = np(1-p) = 25(0.05)(0.95) = 1.1875 \quad \sigma_X = \sqrt{1.1875} = 1.089724$

6. The number of flaws on a magnetic tape produced continuously at a factory follows a Poisson distribution with an average of 0.02 flaws per meter. A standard tape contains 250 meters of magnetic tape.
a) [3] What is the probability that there are at least two flaws in a single tape?

b) [3] What is the probability that there are no flaws in a single tape; that is, a tape is flawless?

c) [5] In a random sample of 25 tapes, what is the probability that at least one of them are flawless?

Solution: Let the random variable X be the # of flaws in a single tape. Then, we know that the distribution of X is $Poisson(250 * 0.02)$, that is $Poisson(5)$.
(a) We are asked for $P(X \geq 2) = 1 - P(X \leq 1) = 1 - 0.040 = 0.96$ (The number 0.040 obtained from the cdf table for Poisson distribution).
(b) We are asked for $P(X = 0) = 0.007$ (from the cdf table for Poisson distribution)
(c) Let the random variable Y be the # of flawless tapes among the 25 randomly selected tapes. Each single tape may or may not be flawless. The probability of observing a flawless tape is what we obtained in part (b), i.e., 0.007. Therefore, the distribution of the random variable Y is $Binomial(25, 0.007)$. The question asks for $P(Y \geq 1) = 1 - P(Y \leq 0) = 1 - P(Y=0) = 1 - \binom{25}{0}(0.007)^0(0.993)^{25} = 1 - (0.993)^{25}$.

7. A box of candy contains 30 pieces. Twenty-six are made of chocolate and four are made of vanilla.

a) [5] Five pieces are selected at random *without replacement*. What is the probability that four of them are chocolates?

b) [5] What would be the answer to part (a) if the five pieces are selected at random *with replacement* (i.e., a selected piece is put back in the box before the next selection is made)?

Solution:
(a) Let Y be the # of chocolate candies in selected in the selected five candies. In this case, Y has $Hypergeometric(30, 26, 5)$. Now,

$$P(Y = 4) = \frac{\binom{26}{4}\binom{4}{1}}{\binom{30}{5}} = 0.419$$

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(b) Let X be the # of chocolate candies in selected in the selected five candies. In this case the distribution of X is $Binomial(5, \frac{26}{30})$. Now,

$$P(X = 4) = \binom{5}{4}(\frac{26}{30})^4(\frac{4}{30})^1 = 0.376$$

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