ECON 2400A 2012 Summer

Assignment 1 Solution

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Q1 (10 points). Suppose we have two products in the market, orange and potato, the demand function and supply function for orange:

$Q_{d1} = 10 - 2P_1 + P_2$

$Q_{s1} = -2 + 3P_1$

the demand function and supply function for potato:

$Q_{d2} = 15 + P_1 - P_2$

$Q_{s2} = -1 + 2P_2$

Find the market clearing (general equilibrium) price and quantities for orange and potato by using the market clearing condition: Demand=Supply.

Solution:

At equilibrium,

$Q_{d1} = Q_{s1}$

$Q_{d2} = Q_{s2}$

i.e.

(1) $10 - 2P_1 + P_2 = -2 + 3P_1$

(2) $15 + P_1 - P_2 = -1 + 2P_2$

Rearrange above equations:

(3) $5P_1 - P_2 - 12 = 0$

(4) $P_1 - 3P_2 + 16 = 0$

From(3), we have

(5) $P_2 = 5P_1 - 12$

Substitute (5) into (4),

$P_1 - 3(5P_1 - 12) + 16 = 0$

$P_1 - 15P_1 + 36 + 16 = 0$

$-14P_1 + 52 = 0$

(6) $P_1^* = \frac{52}{14} = \frac{26}{7}$

Substitute (6) into (5),

(7) $P_2^* = 5P_1^* - 12 = 5 * \frac{26}{7} - 12 = \frac{46}{7}$

Substitute (6) into the supply function of orange,

$Q_1^* = -2 + 3P_1^* = -2 + 3 * \frac{26}{7} = \frac{64}{7}$

substitute (7) into the supply function of potato:

$Q_2^* = -1 + 2P_2^* = -1 + 2 * \frac{46}{7} = \frac{85}{7}$

Answer:

The market clearing price for orange is $\frac{26}{7}$.The market clearing price for potato is $\frac{46}{7}$. Market clearing quantity for orange is $\frac{64}{7}$. The Market cleaning quantity for potato is $\frac{85}{7}$.

Q2 (10 points). Suppose the national-income model:

$Y = C + I_0 + G_0$

$C = a + b(Y - T)(a > 0, 0 < b < 1)$

$T = d + tY(d > 0, 0 < t < 1)$

Where Y is national-income, C is consumption, I_0 is investment, G_0 is government expenditure, T is tax, t is income tax rate, a,b and d are parameters.

i) Which variables are endogenous, which variables are exogenous?

ii) Find equilibrium national-income Y^*, tax T^* and consumption C^*.

iii) Suppose $a = 25, b = 0.3, t = 0.1, d = 50$, $I_0 = 16$ and $G_0 = 14$, calculate Y^*, T^*, and C^*.

Solution:

The national-income model,

(1) $Y = C + I_0 + G_0$

(2) $C = a + b(Y - T)$

(3) $T = d + tY$

i) Endogenous variables: Y, C, T; exogenous variables: I_0, G_0.

ii) Substitute (3) into (2),

(4) $C = a + b(Y - d - tY)$

Substitute (4) into (1),

$Y = a + b(y - d - tY) + I_0 + G_0 = a + by - bd - btY + I_0 + G_0$

Rearrange:

$(1 - b + bt)Y = a - bd + I_0 + g_0$

(5) $Y^* = \frac{a-bd+I_0+G_0}{1-b-bt}$

Substitute (5) into (4),

$C^* = a + b(Y^* - d - tY^*) = a + b(\frac{a-bd+I_0+G_0}{1-b+bt} - d - t\frac{a-bd+I_0+g_0}{1-b+bt})$
$= \frac{a(1-b+bt)+b[(a-bd+I_0+G_0)-d(1-b+bt)-t(a-ad+I_0+G_0]}{1-b+bt}$
$= \frac{a-ab+abt+ab-b^2d+bI_0+bG_0-bd+b^2d-b^2dt-abt+b^2td-btI_0-btG_0}{1-b+bt}$
$= \frac{a-bd+b(1-t)(I_0+G_0)}{1-b+bt}$

Substitute (5) into (3),

$T^* = d + tY^*$
$= d + t\frac{a-bd+I_0+G_0}{1-b+bt}$
$= \frac{d(1-b+bt)+t(a-ad+I_0+G_0)}{1-b+bt}$
$= \frac{d-db+dbt+ta-tbd+tI_0+tG_0}{1-b+bt}$
$= \frac{d(1-b)+t(a+I_0+G_0)}{1-b+bt}$

iii) If a=25, b=0.3, t=0.1, d=50, $I_0 = 16$, $G_0 = 14$, then,

$Y^* = \frac{25-0.3*50+16*14}{1-0.3+0.3*0.1} = \frac{4000}{73}$
$C^* = \frac{25-0.3*50+0.3(1-0.1)(16+14)}{1-0.3+0.3*0.1} = \frac{1810}{73}$
$T^* = \frac{50(1-0.3+0.1(25+16+14)}{1-0.3+0.3*0.1} = \frac{4050}{73}$

Q3 (10 points).Given

A= $\begin{pmatrix} -2 & 3 & 2 \\ 3 & 0 & 3 \\ 4 & 1 & -1 \end{pmatrix}$

B= $\begin{pmatrix} 0 & 4 & 1 \\ -1 & 3 & 2 \\ 4 & -2 & -1 \end{pmatrix}$

C= $\begin{pmatrix} 1 & 0 & 9 \\ 6 & 1 & 1 \\ 2 & -1 & -3 \end{pmatrix}$

Verify that the equation $(ABC)' = C'B'A'$ holds.

Solution:

AB= $\begin{pmatrix} -2 & 3 & 2 \\ 3 & 0 & 3 \\ 4 & 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 4 & 1 \\ -1 & 3 & 2 \\ 4 & -2 & -1 \end{pmatrix}$

$= \begin{pmatrix} (-2)*0+3*1+2*4 & (-2)*4+3*3+2*(-2) & (-2)*1+3*2+2*(-1) \\ 3*0+0*(-1)+3*4 & 3*4+0*3+3*(-2) & 3*1+0*2+3*(-1) \\ 4*0+1*1+(-1)*4 & 4*4+1*3+(-1)*(-2) & 4*1+1*2+(-1)*(-1) \end{pmatrix}$

$$=\begin{pmatrix} 5 & -3 & 2 \\ 12 & 6 & 0 \\ -5 & 21 & 7 \end{pmatrix}$$

$$\text{ABC}=\begin{pmatrix} 5 & -3 & 2 \\ 12 & 6 & 0 \\ -5 & 21 & 7 \end{pmatrix}\begin{pmatrix} 1 & 0 & 9 \\ 6 & 1 & 1 \\ 2 & -1 & -3 \end{pmatrix}$$

$$=\begin{pmatrix} 5*1+(-3)*6+2*2 & 5*0+(-3)*1+2*(-1) & 5*9+(-3)*1+2*(-3) \\ 12*1+6*6+0*2 & 12*0+6*1+0*(-1) & 12*9+6*1+0*(-3) \\ (-5)*1+21*6+7*2 & (-5)*0+21*1+7*(-1) & (-5)*9+21*1+7*(-3) \end{pmatrix}$$

$$=\begin{pmatrix} -9 & -5 & 36 \\ 48 & 6 & 114 \\ 135 & 14 & -45 \end{pmatrix}$$

$$(ABC)'=\begin{pmatrix} -9 & 48 & 135 \\ -5 & 6 & 14 \\ 36 & 114 & -45 \end{pmatrix}$$

$$C'=\begin{pmatrix} 1 & 6 & 2 \\ 0 & 1 & -1 \\ 9 & 1 & -3 \end{pmatrix}$$

$$B'=\begin{pmatrix} 0 & -1 & 4 \\ 4 & 3 & -2 \\ 1 & 2 & -1 \end{pmatrix}$$

$$A'=\begin{pmatrix} -2 & 3 & 4 \\ 3 & 0 & 1 \\ 2 & 3 & -1 \end{pmatrix}$$

$$C'B'=\begin{pmatrix} 1 & 6 & 2 \\ 0 & 1 & -1 \\ 9 & 1 & -3 \end{pmatrix}\begin{pmatrix} 0 & -1 & 4 \\ 4 & 3 & -2 \\ 1 & 2 & -1 \end{pmatrix}$$

$$=\begin{pmatrix} 1*0+6*4+2*1 & 1*(-1)+6*3+2*2 & 1*4+6*(-2)+2*(-1) \\ 0*0+1*4+(-1)*1 & 0*(-1)+1*3+(-1)*2 & 0*4+1*(-2)+(-1)*(-1) \\ 9*0+1*4+(-3)*1 & 9*(-1)+1*3+(-3)*2 & 9*4+1*(-2)+(-3)*(-1) \end{pmatrix}$$

$$=\begin{pmatrix} 26 & 21 & -10 \\ 3 & 1 & 1 \\ 1 & -12 & 37 \end{pmatrix}$$

$C'B'A' = \begin{pmatrix} 26 & 21 & -10 \\ 3 & 1 & 1 \\ 1 & -12 & 37 \end{pmatrix} \begin{pmatrix} -2 & 3 & 4 \\ 3 & 0 & 1 \\ 2 & 3 & -1 \end{pmatrix}$

$= \begin{pmatrix} 26*(-2)+21*3+(-10)*2 & 26*3+21*0+(-10)*3 & 26*4+21*1+(-10)*(-1) \\ 3*(-2)+1*3+(-1)*2 & 3*3+1*0+(-1)*3 & 3*4+181+(-1)*(-1) \\ 1*(-2)+(-12)*3+37*2 & 1*3+(-12)*0+37*3 & 1*4+(-12)*1+37*(-1) \end{pmatrix}$

$= \begin{pmatrix} (-9) & 48 & 135 \\ (-5) & 6 & 14 \\ 36 & 114 & (-45) \end{pmatrix}$

therefore, $(ABC)' = C'B'A'$

Q4 (10 points). Given

A= $\begin{pmatrix} 0 & 4 \\ -1 & 3 \end{pmatrix}$

B= $\begin{pmatrix} 3 & -8 \\ 0 & 1 \end{pmatrix}$

C= $\begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix}$

Verify that the equation $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$ holds.

Solution:

AB= $\begin{pmatrix} 0 & 4 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 3 & -8 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0*3+4*0 & 0*(-8)+4*1 \\ (-1)*3+3*0 & (-1)*(-8)+3*1 \end{pmatrix} = \begin{pmatrix} 0 & 4 \\ -3 & 11 \end{pmatrix}$

ABC= $\begin{pmatrix} 0 & 4 \\ -3 & 11 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 6 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0*1+4*6 & 0*0+4*1 \\ (-3)*1+11*6 & (-3)*0+11*1 \end{pmatrix} = \begin{pmatrix} 24 & 4 \\ 63 & 11 \end{pmatrix}$

$|ABC| = (24*11-63*4) = 12$

$(ABC)^{-1} = \frac{1}{12} \begin{pmatrix} 11 & -4 \\ -63 & 24 \end{pmatrix} = \begin{pmatrix} \frac{11}{12} & -\frac{1}{3} \\ -\frac{21}{4} & 2 \end{pmatrix}$

$|A| = 0*3-4*(-1) = 4$

$A^{-1} = \frac{1}{4} \begin{pmatrix} 3 & -4 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{3}{4} & -1 \\ \frac{1}{4} & 0 \end{pmatrix}$

$|B| = 3*1-(-8)*0 = 3$

B^{-1}=$\frac{1}{3}\begin{pmatrix} 1 & 8 \\ 0 & 3 \end{pmatrix}=\begin{pmatrix} \frac{1}{3} & \frac{8}{3} \\ 0 & 1 \end{pmatrix}$

$|C| = 1 * 1 - 0 * 6 = 1$

$C^{-1} = \begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}$

$C^{-1}B^{-1}$=$\begin{pmatrix} 1 & 0 \\ -6 & 1 \end{pmatrix}\begin{pmatrix} \frac{1}{3} & \frac{8}{3} \\ 0 & 1 \end{pmatrix}$

=$\begin{pmatrix} \frac{1}{3} & \frac{8}{3} \\ -2 & -15 \end{pmatrix}$

$C^{-1}B^{-1}A^{-1}$=$\begin{pmatrix} \frac{1}{3} & \frac{8}{3} \\ -2 & -15 \end{pmatrix}\begin{pmatrix} \frac{3}{4} & -1 \\ \frac{1}{4} & 0 \end{pmatrix}=\begin{pmatrix} \frac{11}{12} & -\frac{1}{3} \\ -\frac{21}{4} & 2 \end{pmatrix}$

therefore, $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

Q5 (10 points). Test the nonsingularity of the following matrix by finding its rank and determinant.

A= $\begin{pmatrix} 7 & -1 & 0 \\ 1 & 1 & 4 \\ 13 & -3 & -4 \end{pmatrix}$

Solution:

$|A| = 7*1*(-4)+(-1)*4*13+0*1*(-3)-0*1*13-4*(-3)*7-(-4)*1*(-1)$
=0

Let A=$\begin{pmatrix} 7 & -1 & 0 \\ 1 & 1 & 4 \\ 13 & -3 & -4 \end{pmatrix}=\begin{pmatrix} V_1' \\ V_2' \\ V_3' \end{pmatrix}$

$2V_1' - V_2' = [2 * 7 2 * (-1) - 1 2 * 0 - 4] = [13 - 3 - 4] = V_3'$

i.e. The third row is expressed as a linear function of the first two rows.

A= $\begin{pmatrix} 7 & -1 & 0 \\ 1 & 1 & 4 \\ 13 & -3 & -4 \end{pmatrix}$ A= $\begin{pmatrix} 7 & -1 & 0 \\ 1 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$ (By $2V_1' - V_2$)

Rank(A)= $2 < 3$ **therefore, A is not non-singular, but singular.**

Q6 (10 points). Use matrix inversion and Cramer's rule to solve the following equation system:

$$4x + 3y - 2z = 1$$
$$x + 2y = 6$$
$$3x + z = 4$$

Solution:

Let A= $\begin{pmatrix} 4 & 3 & -2 \\ 1 & 2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$

X= $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$

d= $\begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$

Then AX=d,

i) Use inverse matrix method:

X=A^{-1}d

$|A| = 4*2*1 + 3*0*3 + (-2)*1*0 - (-2)*2*3 - 3*1*1 - 4*0*0 = 17$

$|C_{11}| = 2, |C_{21}| = -3, |C_{31}| = 4,\ |C_{12}| = -1, |C_{22}| = 10, |C_{32}| = -2,$

$|C_{13}| = -6, |C_{23}| = 9, |C_{33}| = 5$

A^{-1}=$\frac{1}{|A|}\begin{pmatrix} |C_{11} & |C_{21}| & |C_{31}| \\ |C_{12}| & |C_{22}| & |C_{32}| \\ |C_{13}| & |C_{23}| & |C_{33}| \end{pmatrix}$

=$\frac{1}{17}\begin{pmatrix} 2 & -3 & 4 \\ -1 & 10 & -2 \\ -6 & 9 & 5 \end{pmatrix}$

X=$\frac{1}{17}\begin{pmatrix} 2 & -3 & 4 \\ -1 & 10 & -2 \\ -6 & 9 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ 6 \\ 4 \end{pmatrix}$

=$\frac{1}{17}\begin{pmatrix} 0 \\ 51 \\ 68 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix}$

i.e. $x^* = 0$

$y^* = 3$

$z^* = 4$

ii) Use Cramer's Rule:

$A_1 = \begin{pmatrix} 1 & 3 & -2 \\ 6 & 2 & 0 \\ 4 & 0 & 1 \end{pmatrix}$

$|A_1| = 1*2*1 + 6*0*(-2) + 3*0*4 - (-2)*2*4 - 3*6*1 - 1*0*0 = 0$

$A_2 = \begin{pmatrix} 4 & 1 & -2 \\ 1 & 6 & 0 \\ 3 & 4 & 1 \end{pmatrix}$

$|A_2| = 4*6*1 + 1*4*(-2) + 1*0*3 - (-2)*6*3 - 1*1*1 - 4*4*0 = 51$

$A_3 = \begin{pmatrix} 4 & 3 & 1 \\ 1 & 2 & 6 \\ 3 & 0 & 4 \end{pmatrix}$

$|A_3| = 4*2*4 + 3*6*3 + 1*0*1 - 1*2*3 - 3*1*4 - 6*0*4 = 68$

So,

$x^* = \frac{|A_1|}{|A|} = \frac{0}{17} = 0$

$y^* = \frac{|A_2|}{|A|} = \frac{51}{17} = 3$

$z^* = \frac{|A_3|}{|A|} = \frac{68}{17} = 4$

Q7 (10 points). Solve the following inequalities:

$(a) 2x - 1 < 6x + 5$

$(b) |2x + 3| < 5$

Solution:

(a) $6x - 2x > -1 - 5$

$4x > -6$

$x > -\frac{2}{3}$

(b)$-5 < 2x + 3 < 5$

$-5 - 3 < 2x + 3 < 5 - 3$

$-8 < 2x < 2$

$-4 < x < 1$

Q8 (10 points).

(a) Find the limit of the following function as x approaches to 4.

$y = f(x) = \frac{x^2-9x+20}{x-4}$

(b) Check whether the limit of the following function exists as x approaches to $-\frac{5}{3}$. Is the function differentiable at $x = -\frac{5}{3}$? Why?

$y = f(x) = |3x + 5| + 6$

Solution:

(a)

$$\lim_{x\to 4} y = \lim_{x\to 4}\frac{x^2-9x+20}{x-4} = \lim_{x\to 4}\frac{(x-4)(x-5)}{x-4} = \lim_{x\to 4}(x-5) = -1$$

(b) Let 3x+5=0

$x = -\frac{5}{3}$, $f(-\frac{5}{3}) = 6$.

$if x > -\frac{5}{3}, then|3x+5| = 3x+5$

$if x < -\frac{5}{3}, then|3x+5| = -3x-5$

$$\lim_{x\to -\frac{5}{3}^+} y = \lim_{x\to -\frac{5}{3}^+}(3x+5+6) = 6$$

$$\lim_{x\to -\frac{5}{3}^-} y = \lim_{x\to -\frac{5}{3}^-}(-3x-5+6) = 6$$

$$\lim_{x\to -\frac{5}{3}^+} y = \lim_{x\to -\frac{5}{3}^-}(y) == f(-\frac{5}{3}) = 6$$

Therefore, the limit of the function exists as $x \to -\frac{5}{3}$.

To check the differentiability, we need check the limit of the difference quotient:

$$\lim_{x\to -\frac{5}{3}}\frac{f(x)-f(-\frac{5}{3})}{x+\frac{5}{3}} = \lim_{x\to -\frac{5}{3}}\frac{|3x+5|+6-6}{x+\frac{5}{3}} = \lim_{x\to -\frac{5}{3}}\frac{|3x+5|}{x+\frac{5}{3}}$$

$$\lim_{x\to -\frac{5}{3}^+}\frac{|3x+5|}{x+\frac{5}{3}} = \lim_{x\to -\frac{5}{3}^+}\frac{3x+5}{x+\frac{5}{3}} = 3$$

$$\lim_{x\to -\frac{5}{3}^-}\frac{|3x+5|}{x+\frac{5}{3}} = \lim_{x\to -\frac{5}{3}^-}\frac{-(3x+5)}{x+\frac{5}{3}} = -3$$

Because

$$\lim_{x\to -\frac{5}{3}^-}\frac{|3x+5|}{x+\frac{5}{3}} \neq \lim_{x\to -\frac{5}{3}^+}\frac{|3x+5|}{x+\frac{5}{3}}$$

So the function is not differentiable at $x = -\frac{5}{3}$.

Q9 (10 points).

(a) Find the derivative of the following function.

$y = f(x) = \frac{ax^2+b}{cx+d}$

(b) Find the marginal and average functions for the total-cost function:

$C = 3Q^2 + 7Q + 12$

Solution:

Use Quotient Rule:

(a) $\frac{d}{dx}\frac{z(x)}{g(x)} = \frac{z^{'}(x)g(x)-z(x)g^{'}(x)}{g^2(x)}$

Here $z(x) = ax^2 + b$, and $g(x) = cx + d$

$z^{'}(x) = 2ax$

$g^{'}(x) = c$

Therefore, $\frac{dy}{dx} = \frac{2ax(cx+d)-(ax^2+b)c}{(cx+d)^2}$

$= \frac{acx^2+2adx-bc}{(cx+d)^2}$

(b) Marginal cost $MC \equiv \frac{dc(Q)}{d(Q)}$

Average cost $AC \equiv \frac{c(Q)}{Q}$

$MC = 6Q + 7$

$AC = \frac{3Q^2+7Q+12}{Q}$

Q10 (10 points).

(a) Find the instantaneous rate of growth (see the definition on Page 286) of the following function:

$y = 2^t(t^2)$

(b) Given the demand function $Q_d = \frac{k}{P^n}$, where k and n are positive constants. Find the point elasticity of demand (see the definition on Page 289).

Solution:

(a) Instantaneous Rate of growth

$\gamma_y \equiv \frac{\frac{dy}{dt}}{y}$

$\frac{dy}{dt} = 2^t t^2 ln2 + 2t2^t$

$\gamma_y = \frac{2^t t^2 ln2 + 2t*2^t}{2^t t^2}$

$= \frac{tln2+2}{t}$

(b) Point elasticity of demand:

$\varepsilon_d \equiv \frac{d(lnQ_d)}{dlnP}$

$= \frac{P}{Q_d}\frac{dQ_d}{dP}$

$\frac{dQ_d}{dP} = -\frac{nk}{P^{n+1}}$

$\varepsilon_d = -\frac{Pnk}{P^n} = -n$

(Or, from

$Q_d = \frac{k}{p^n}$

we have,

$lnQ_d = lnk - nlnp$

$\varepsilon_d = \frac{dlnQ_d}{dlnp} = -n)$