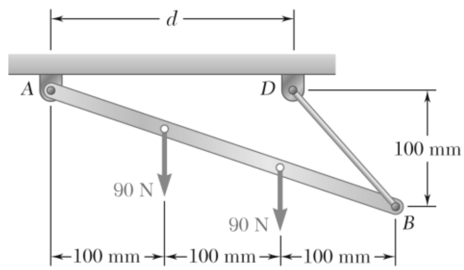
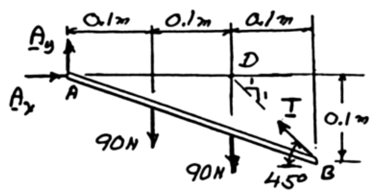


PROBLEM 4.27

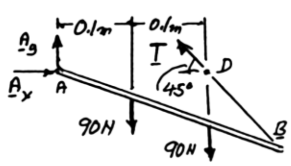
A rod *AB* hinged at *A* and attached at *B* to cable *BD* supports the loads shown. Knowing that $d = 200$ mm, determine (*a*) the tension in cable *BD*, (*b*) the reaction at *A*.

SOLUTION

Free-Body Diagram:



(*a*) Move **T** along *BD* until it acts at Point *D*.



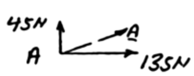
$$+\circlearrowleft \Sigma M_A = 0: \quad (T \sin 45°)(0.2 \text{ m}) + (90 \text{ N})(0.1 \text{ m}) + (90 \text{ N})(0.2 \text{ m}) = 0$$

$$T = 190.919 \text{ N} \qquad T = 190.9 \text{ N} \blacktriangleleft$$

(*b*)

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x - (190.919 \text{ N})\cos 45° = 0$$

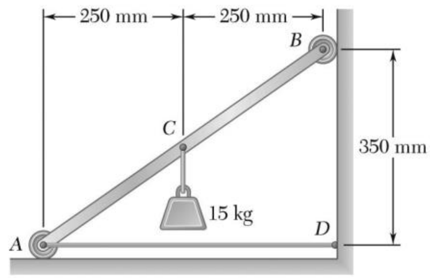
$$A_x = +135.0 \text{ N} \qquad \mathbf{A}_x = 135.0 \text{ N} \longrightarrow$$



$$+\uparrow \Sigma F_y = 0: \quad A_y - 90 \text{ N} - 90 \text{ N} + (190.919 \text{ N})\sin 45° = 0$$

$$A_y = +45.0 \text{ N} \qquad \mathbf{A}_y = 45.0 \text{ N} \uparrow$$

$$\mathbf{A} = 142.3 \text{ N} \measuredangle 18.43° \blacktriangleleft$$

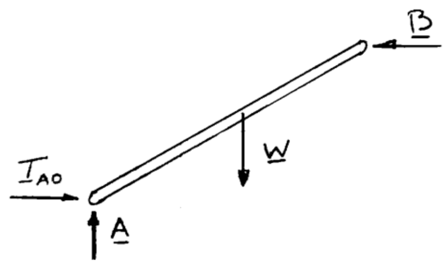


PROBLEM 4.36

A light bar AB supports a 15-kg block at its midpoint C. Rollers at A and B rest against frictionless surfaces, and a horizontal cable AD is attached at A. Determine (a) the tension in cable AD, (b) the reactions at A and B.

SOLUTION

Free-Body Diagram:



$$W = (15\text{ kg})(9.81\text{ m/s}^2)$$
$$= 147.150\text{ N}$$

(a) $\xrightarrow{+} \Sigma F_x = 0$: $T_{AD} - 105.107\text{ N} = 0$

$$T_{AD} = 105.1\text{ N} \blacktriangleleft$$

(b) $+\uparrow \Sigma F_y = 0$: $A - W = 0$

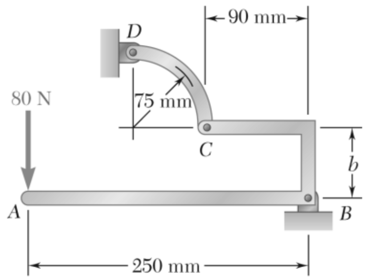
$A - 147.150\text{ N} = 0$

$$\mathbf{A} = 147.2\text{ N}\uparrow \blacktriangleleft$$

$+\circlearrowleft \Sigma M_A = 0$: $B(350\text{ mm}) - (147.150\text{ N})(250\text{ mm}) = 0$

$B = 105.107\text{ N}$

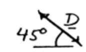
$$\mathbf{B} = 105.1\text{ N} \leftarrow \blacktriangleleft$$



PROBLEM 4.67

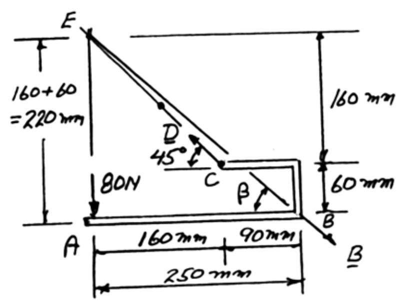
Determine the reactions at B and D when $b = 60$ mm.

SOLUTION

Since *CD* is a two-force member, the line of action of reaction at *D* must pass through Points *C* and *D*. 

Free-Body Diagram:

(Three-force body)

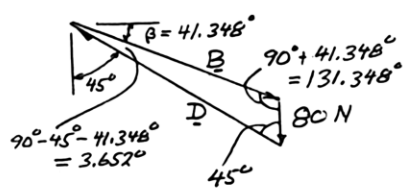


Reaction at *B* must pass through *E*, where the reaction at *D* and the 80-N force intersect.

$$\tan\beta = \frac{220 \text{ mm}}{250 \text{ mm}}$$

$$\beta = 41.348°$$

<u>Force triangle</u>



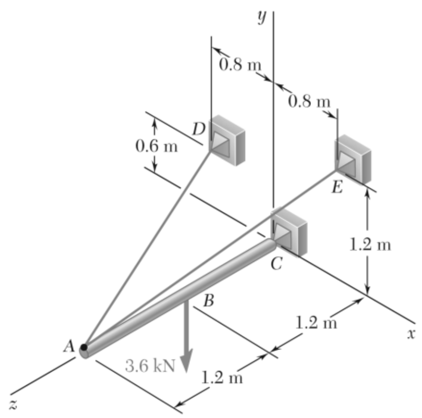
Law of sines:

$$\frac{80 \text{ N}}{\sin 3.652°} = \frac{B}{\sin 45°} = \frac{D}{\sin 131.348°}$$

$$B = 888.0 \text{ N}$$

$$D = 942.8 \text{ N}$$

B = 888 N ↖ 41.3°   **D** = 943 N ↘ 45.0° ◀

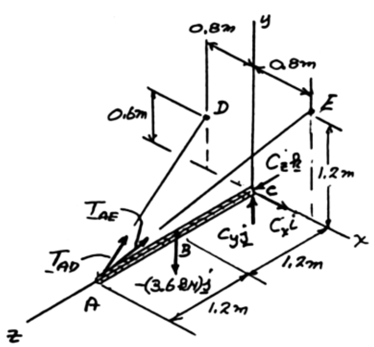


PROBLEM 4.105

A 2.4-m boom is held by a ball-and-socket joint at C and by two cables AD and AE. Determine the tension in each cable and the reaction at C.

SOLUTION

Free-Body Diagram: Five unknowns and six equations of equilibrium, but equilibrium is maintained ($\Sigma M_{AC} = 0$).



$$\mathbf{r}_B = 1.2\mathbf{k}$$
$$\mathbf{r}_A = 2.4\mathbf{k}$$
$$\overrightarrow{AD} = -0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k} \qquad AD = 2.6 \text{ m}$$
$$\overrightarrow{AE} = 0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k} \qquad AE = 2.8 \text{ m}$$

$$T_{AD} = \frac{\overrightarrow{AD}}{AD} = \frac{T_{AD}}{2.6}(-0.8\mathbf{i} + 0.6\mathbf{j} - 2.4\mathbf{k})$$
$$T_{AE} = \frac{\overrightarrow{AE}}{AE} = \frac{T_{AE}}{2.8}(0.8\mathbf{i} + 1.2\mathbf{j} - 2.4\mathbf{k})$$

$$\Sigma M_C = 0: \quad \mathbf{r}_A \times \mathbf{T}_{AD} + \mathbf{r}_A \times \mathbf{T}_{AE} + \mathbf{r}_B \times (-3 \text{ kN})\mathbf{j} = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ -0.8 & 0.6 & -2.4 \end{vmatrix} \frac{T_{AD}}{2.6} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 2.4 \\ 0.8 & 1.2 & -2.4 \end{vmatrix} \frac{T_{AE}}{2.8} + 1.2\mathbf{k} \times (-3.6 \text{ kN})\mathbf{j} = 0$$

Equate coefficients of unit vectors to zero:

$$\mathbf{i}: \quad -0.55385T_{AD} - 1.02857T_{AE} + 4.32 = 0 \qquad (1)$$
$$\mathbf{j}: \quad -0.73846T_{AD} + 0.68671T_{AE} = 0$$

$$T_{AD} = 0.92857T_{AE} \qquad (2)$$

From Eq. (1):

$$-0.55385(0.92857)T_{AE} - 1.02857T_{AE} + 4.32 = 0$$

$$1.54286T_{AE} = 4.32$$
$$T_{AE} = 2.800 \text{ kN}$$

$T_{AE} = 2.80 \text{ kN}$ ◀

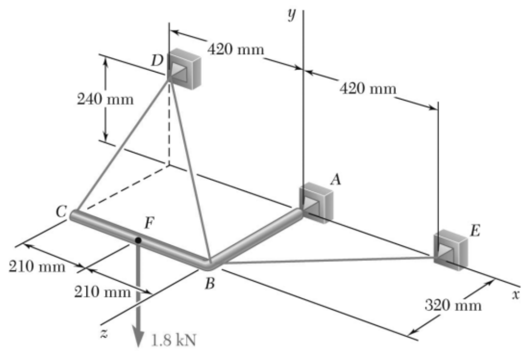
From Eq. (2): $T_{AD} = 0.92857(2.80) = 2.600\text{ kN}$ $\qquad T_{AD} = 2.60\text{ kN}$ ◀

$$\Sigma F_x = 0: \quad C_x - \frac{0.8}{2.6}(2.6\text{ kN}) + \frac{0.8}{2.8}(2.8\text{ kN}) = 0 \qquad C_x = 0$$

$$\Sigma F_y = 0: \quad C_y + \frac{0.6}{2.6}(2.6\text{ kN}) + \frac{1.2}{2.8}(2.8\text{ kN}) - (3.6\text{ kN}) = 0 \qquad C_y = 1.800\text{ kN}$$

$$\Sigma F_z = 0: \quad C_z - \frac{2.4}{2.6}(2.6\text{ kN}) - \frac{2.4}{2.8}(2.8\text{ kN}) = 0 \qquad C_z = 4.80\text{ kN}$$

$\mathbf{C} = (1.800\text{ kN})\mathbf{j} + (4.80\text{ kN})\mathbf{k}$ ◀

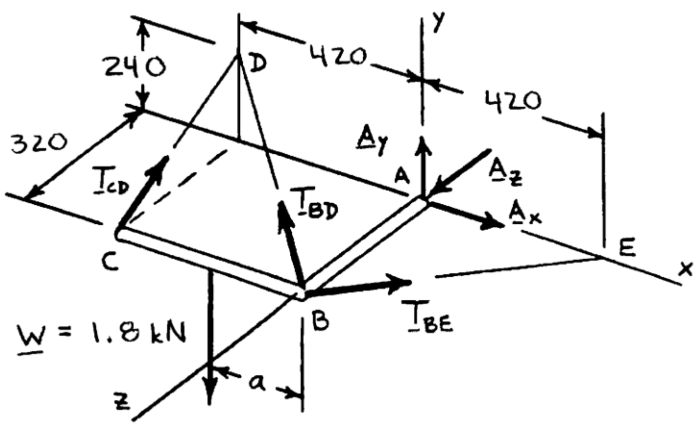


PROBLEM 4.125

The rigid L-shaped member *ABC* is supported by a ball-and-socket joint at *A* and by three cables. If a 1.8-kN load is applied at *F*, determine the tension in each cable.

SOLUTION

Free-Body Diagram: Dimensions in mm



In this problem: $a = 210$ mm

We have

$$\overrightarrow{CD} = (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad CD = 400\text{ mm}$$

$$\overrightarrow{BD} = -(420\text{ mm})\mathbf{i} + (240\text{ mm})\mathbf{j} - (320\text{ mm})\mathbf{k} \quad BD = 580\text{ mm}$$

$$\overrightarrow{BE} = (420\text{ mm})\mathbf{i} - (320\text{ mm})\mathbf{k} \quad BE = 528.02\text{ mm}$$

Thus,

$$T_{CD} = T_{CD}\frac{\overrightarrow{CD}}{CD} = T_{CD}(0.6\mathbf{j} - 0.8\mathbf{k})$$

$$T_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} = T_{BD}(-0.72414\mathbf{i} + 0.41379\mathbf{j} - 0.55172\mathbf{k})$$

$$T_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} = T_{BE}(0.79542\mathbf{i} - 0.60604\mathbf{k})$$

$$\Sigma\mathbf{M}_A = 0\text{:} \quad (\mathbf{r}_C \times \mathbf{T}_{CD}) + (\mathbf{r}_B \times \mathbf{T}_{BD}) + (\mathbf{r}_B \times \mathbf{T}_{BE}) + (\mathbf{r}_W \times \mathbf{W}) = 0$$

Noting that

$$\mathbf{r}_C = -(420 \text{ mm})\mathbf{i} + (320 \text{ mm})\mathbf{k}$$
$$\mathbf{r}_B = (320 \text{ mm})\mathbf{k}$$
$$\mathbf{r}_W = -a\mathbf{i} + (320 \text{ mm})\mathbf{k}$$

and using determinants, we write

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -420 & 0 & 320 \\ 0 & 0.6 & -0.8 \end{vmatrix} T_{CD} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 320 \\ -0.72414 & 0.41379 & -0.55172 \end{vmatrix} T_{BD}$$
$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 320 \\ 0.79542 & 0 & -0.60604 \end{vmatrix} T_{BE} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -a & 0 & 320 \\ 0 & -1.8 & 0 \end{vmatrix} = 0$$

Equating to zero the coefficients of the unit vectors,

$$\mathbf{i}: \quad -192T_{CD} - 132.413T_{BD} + 576 = 0 \tag{1}$$

$$\mathbf{j}: \quad -336T_{CD} - 231.72T_{BD} + 254.53T_{BE} = 0 \tag{2}$$

$$\mathbf{k}: \quad -252T_{CD} + 1.8a = 0 \tag{3}$$

Recalling that $a = 210$ mm, Eq. (3) yields

$$T_{CD} = \frac{1.8(210)}{252} = 1.500 \text{ kN} \qquad T_{CD} = 1.500 \text{ kN} \blacktriangleleft$$

From Eq. (1): $-192(1.5) - 132.413T_{BD} + 576 = 0$

$$T_{BD} = 2.1751 \text{ kN} \qquad T_{BD} = 2.18 \text{ kN} \blacktriangleleft$$

From Eq. (2): $-336(1.5) - 231.72(2.1751) + 254.53T_{BE} = 0$

$$T_{BE} = 3.9603 \text{ kN} \qquad T_{BE} = 3.96 \text{ kN} \blacktriangleleft$$