



COURSE: CEG2136
Computer Architecture I
SEMESTER: Fall 2016

Assignment 2 Solutions

Q1. In a signed binary system of 8 bits ($N=8$):

a) Find the 2's complement representation of the following signed numbers :

$$(+63)_{10} = (00111111)_2$$

63/2	remainder		
31	1	lsb	2^0
15	1		2^1
7	1		2^2
3	1		2^3
1	1		2^4
0	1		2^5

	Sign	2^6	2^5	2^4	2^3	2^2	2^1	2^0
$+63=$	0	0	1	1	1	1	1	1

Or observe that $63 = 32+16+8+4+2+1$

$$(-115)_{10} = (10001101)_2$$

115/2	remainder		
57	1	lsb	2^0
28	1		2^1
14	0		2^2
7	0		2^3
3	1		2^4
1	1		2^5
0	1	msb	2^6

	Sign	2^6	2^5	2^4	2^3	2^2	2^1	2^0
$+115=$	0	1	1	1	0	0	1	1

$-115 = 2$'s complement of $(01110011) = 10001101$

- b) Find the 2's complement of the following signed numbers and give your results in decimal, too: $(-63)_{10}$ and $(+115)_{10}$

The 2's complement representation of $(-63)_{10} = (11000001)_2$
 \Rightarrow Do the 2's complement of $(-63)_{10}$ as required in the question
 $=$ 2's complement of $(11000001)_2 = (00111111)_2$ = which is the 2's complement representation of $(+63)_{10}$ (which in fact is the result that we expected)
 The 2's complement representation of $(+115)_{10} = (01110011)$
 \Rightarrow Do the 2's complement of $(+115)_{10}$ = 2's complement of $(01110011) = (10001101)_2$ = 2's complement representation of $(-115)_{10}$

- c) Perform the following arithmetic operations using the signed-2's complement representation and provide your results in decimal (including intermediary steps), as well:

- 1) Say $A = (+115)_{10}$ and $B = (-63)_{10}$
 $S = A + B = (+115)_{10} + (-63)_{10}$

	2's complement representation									Base 10
	CY ₈	CY ₇	CY ₆	CY ₅	CY ₄	CY ₃	CY ₂	CY ₁	CY ₀	
CY:	1	1	0	0	0	0	1	1	0	
A +		0	1	1	1	0	0	1	1	115 ₁₀ ⁺
<u>B</u>		1	1	0	0	0	0	0	1	<u>-63₁₀</u>
S=A+B		0	0	1	1	0	1	0	0	52 ₁₀

CY₈ = CY₇ = 1 \Rightarrow NO overflow

- 2) Say $C = (-115)_{10}$ and $D = (-63)_{10}$
 $X = C - D = (-115)_{10} - (-63)_{10} = (-115)_{10} + [- (-63)]_{10} = (-115)_{10} + (+63)_{10}$

	2's complement representation									Base 10
	CY ₈	CY ₇	CY ₆	CY ₅	CY ₄	CY ₃	CY ₂	CY ₁	CY ₀	
CY:	0	0	1	1	1	1	1	1	0	
C+		1	0	0	0	1	1	0	1	-115 ₁₀
<u>-D</u>		0	0	1	1	1	1	1	1	<u>63₁₀</u>
X=C-D		1	1	0	0	1	1	0	0	-52 ₁₀

As the msb of $X = C - D$ is 1 $\Rightarrow X$ is a negative number ($X = -y$) with a magnitude
 $y = -(-y) = -(X) = 2$'s complement of $(X) = 2$'s complement of $(11001100) = 00110100 = 52_{10}$
 $\Rightarrow X = -y = -52_{10}$
CY₈ = CY₇ = 0 \Rightarrow NO overflow

Q2. a) Identify the decimal number which is represented next with 32-bit in the IEEE 754 standard:

(1 **10001011 111010000000000000000000**) = (?)₁₀
 Sign = 1 : \Rightarrow negative ; C = 139, hence Exp = C-127 = 12
 1.M = 1.**111101** (where M = mantissa)
 $-1.111101 * 2^{12} = -1111010000000 = (-7808)_{10}$

b) Represent (221.390625) in the IEEE 754 standard with 32 bits.
 $(221.390625)_{10} = (11011101.011001)_2 = (1.1011101011001) * 2^7$
 Sign = 0 ; Exp = 7 ; C = 127 + 7 = 134 = (10000110)₂
 M = **1011101011001**
 $(221.390625)_{10} = (0 10000110 101110101100100000000000)_{IEEE754}$
 3.1.

$$(101110)_2 = 32 + 8 + 4 + 2 = 46$$

$$(1110101)_2 = 64 + 32 + 16 + 4 + 1 = 117$$

$$(110110100)_2 = 256 + 128 + 32 + 16 + 4 = 436$$

3.3.

$$(1231)_{10} = 1024 + 128 + 64 + 15 = 2^{10} + 2^7 + 2^6 + 2^3 + 2^2 + 2^1 + 1 = (10011001111)_2$$

$$(673)_{10} = 512 + 128 + 32 + 1 = 2^9 + 2^7 + 2^5 + 1 = (1010100001)_2$$

$$(1998)_{10} = 1024 + 512 + 256 + 128 + 64 + 8 + 4 + 2$$

$$= 2^{10} + 2^9 + 2^8 + 2^7 + 2^6 + 2^3 + 2^2 + 2^1 = (11111001110)_2$$

3.4

$$(7562)_{10} = (16612)_8$$

$$(1938)_{10} = (792)_{16}$$

$$(175)_{10} = (10101111)_2$$

3.5

$$(F3A7C2)_{16} = (1111\ 0011\ 1010\ 0111\ 1100\ 0010)_2$$

$$= (74723702)_8$$

3.8

$$(295)_{10} = 256 + 32 + 7 = (100100111)_2$$

(a) 0000 0000 0000 0001 0010 0111

(b) 0000 0000 0000 0010 1001 0101

(c) 10110010 00111001 00110101

3.10. Decode the following ASCII code:

1001010	1001111	1001000	1001110	0100000	1000100	1001111	1000101
4A	4F	48	4E	20	44	4F	45
J	O	H	N		D	O	E

3.13

	10101110;	10000001	10000000	00000001	00000000.
1's compl	01010001	01111110	01111111;	11111110	11111111
2's compl	01010010	01111111	10000000	11111111	00000000

3.16

$+42 = 0101010$	$+13 = 0001101$
$-42 = 1010110$	$-13 = 1110011$
$(+42) 0101010$	$(-42) 1010110$
$(-13) 1110011$	$(+13) 0001101$
$(+29) 0011101$	$(-29) 1100011$