COMP 232 Mathematics for Computer Science
Fall 2015
Midterm Exam

Name: ______________________ Total Points:

ID: ______________________ ________ / 24

Instructions. *This is a closed book exam. The only allowed tool is an ENCS approved calculator. Provide all answers in this booklet. Use pen, not pencil. Do not detach any pages from this exam!*

($1^{\text{pt}}_{\text{ea.}}$) **1.** For each of the following propositional sentences, state whether or not it is a tautology. [ ] 5 pts

(a) $((p \vee q) \wedge (q \vee r)) \leftrightarrow (q \vee r)$

☑ Not tautology | ☐ Tautology | ☐ Don't know!

(b) $(p \to q) \to (q \to p)$

☐ Tautology | ☑ Not a tautology | ☐ Don't know!

(c) $((p \to q) \wedge (q \to p)) \leftrightarrow (p \leftrightarrow q)$

☐ Not a tautology | ☑ Tautology | ☐ Don't know!

(d) $((p \to q) \wedge (\neg r \to \neg q)) \to (p \to (q \wedge r))$

☐ Not a tautology | ☑ Tautology | ☐ Don't know!

(e) $(p \to (q \to r)) \leftrightarrow ((p \to q) \to r)$

☑ Not a tautology | ☐ Tautology | ☐ Don't know!

[ ] 5 pts

(1pt ea.) **2.** Suppose $P(x, y)$ is a predicate and the Universe of Discourse for variables x and y is $\{1, 2, 3\}$. Suppose $P(1, 3), P(2, 1), P(2, 2), P(2, 3), P(3, 1), P(3, 2)$ are true, and $P(x, y)$ is false otherwise. Determine whether the following statements are true.

5 pts

(a) $\forall x \exists y P(x, y)$

☑ True ☐ False ☐ Don't know!

(b) $\exists x \forall y P(x, y)$

☑ True ☐ False ☐ Don't know!

(c) $\neg \Big[\exists x \exists y \Big(P(x, y) \wedge \neg P(y, x) \Big) \Big]$

☐ True ☑ False ☐ Don't know!

(d) $\forall x \exists x \Big(P(x, y) \rightarrow P(y, x) \Big)$

☑ True ☐ False ☐ Don't know!

(e) $\forall y \exists x \Big(x \leq y \ \wedge \ P(x, y) \Big)$

☐ True ☑ False ☐ Don't know!

5 pts

(6pts) **3.** In the table below, construct a proof of the equivalence

$$\Big(\neg(p \to q) \to p\Big) \equiv T$$

6 pts

Step		Law applied
	$\neg(p \to q) \to p$	Assumption
$\equiv$	$\neg(\neg(p \to q)) \vee p$	Law for conditional
$\equiv$	$(p \to q) \vee p$	Double negation
$\equiv$	$(\neg p \vee q) \vee p$	Law for conditional
$\equiv$	$p \vee (\neg p \vee q)$	Commutativity
$\equiv$	$(p \vee \neg p) \vee q$	Associativity
$\equiv$	$T \vee q$	Law of excluded middle
$\equiv$	$q \vee T$	Commutativity
$\equiv$	T	Domination

6 pts

(2pts ea.) **4.** (a) Consider the assertion "*If x and y are odd integers, then $x+y$ is even.*" In the box below, give a direct proof of the assertion.

8 pts

Solution:

x is odd $\Rightarrow$ $x = 2k+1$, for some $k \in \mathbb{Z}$.
y is odd $\Rightarrow$ $y = 2\ell+1$, for some $\ell \in \mathbb{Z}$.

$\Rightarrow x+y = 2k+1+2\ell+1 = 2k+2\ell+2 = 2(k+\ell+1) \Rightarrow$ $x+y$ is even.

(b) Consider the assertion "*For every integer n, if n^5+7 is even, then n is odd.*" In the box below, give an indirect proof of the assertion.

Solution:

n is even $\Rightarrow n = 2k$, for some $k \in \mathbb{Z}$.

$\Rightarrow n^5+7 = 32k^5+7 = 32k^5+6+1 = 2(16k^5+3)+1 \Rightarrow n^5+7$ is odd

(c) Consider the assertion "*If x is rational and y is irrational, then $x-y$ is irrational.*" In the box below, give a proof by contradiction of the assertion.

Solution:

Counterassumption: x is rational, y is irrational, and $x-y$ is rational.

x is rational $\Rightarrow x = \frac{a}{b}$, where $a, b \in \mathbb{Z}$ and $b \neq 0$.
$x-y$ is rational $\Rightarrow x-y = \frac{c}{d}$, where $c, d \in \mathbb{Z}$ and $d \neq 0$.

$\Rightarrow -y = (x-y) - x = \frac{c}{d} - \frac{a}{b} = \frac{cb-ad}{db} \Rightarrow y = \frac{ad-cb}{db}$

Since $b \neq 0$ and $d \neq 0$ and $y = \frac{ad-cb}{db} \Rightarrow y$ is rational; contradiction!

(d) Consider the assertion "*For all integers n, it holds that n^2+n is even .*" In the box below, give a proof by cases of the assertion.

Solution:

- *Case 1:* n is even.

$\Rightarrow n = 2k$, for some $k \in \mathbb{Z} \Rightarrow n^2+n = 4k^2+2k = 2(2k^2+k)$

$\Rightarrow n^2+n$ is even

- *Case 2:* n is odd.

$\Rightarrow n = 2k+1$, for some $k \in \mathbb{Z}$

$\Rightarrow n^2+n = (2k+1)^2+(2k+1) = (4k^2+4k+1)+(2k+1) = 4k^2+6k+2$
$\Rightarrow n^2+n = 2(2k^2+3k+1)$

$\Rightarrow n^2+n$ is even

... — End of Exam — ...

8 pts