Department of Economics

Carleton University

ECON 2030B – Intermediate Microeconomics II: Consumers and General Equilibrium

ASSIGNMENT 2 - Due Feb 5 at the beginning of the lecture

1. (20 points) By observing an individual's behavior in the situations outlined below, determine the relevant income elasticities of demand for each good, that is, whether each good is normal, neutral or inferior. If you cannot determine whether a good is normal, neutral or inferior, what additional information do you need?

   (a) (5 points) Bill spends all his income on books and coffee. He finds \$20 while rummaging through a used paperback bin at the bookstore. He immediately buys a new hardcover book of poetry.

   Books are a normal good since his consumption of books increases with income. Coffee is a neutral good since consumption of coffee stayed the same when income increased.

   (b) (5 points) Bill loses \$10 he was going to use to buy a double espresso. He decides to sell his new book at a discount to a friend and use the money to buy coffee.

   When Bill;s income decreased by \$10 he decided to own fewer books, so books are a normal good. Coffee appears to be a neutral good because Bill's purchase of the double espresso did not change as his income changed.

   (c) (5 points) Being bohemian becomes the latest trend. As a result, coffee and book prices rise by 25%. Bill lowers his consumption of both goods by the same percentage.

   Books and coffee are both normal goods because Bill's response to a decline in real income is to decrease consumption of both goods. In addition, the income elasticities for both goods are the same because Bill reduces consumption of both by the same percentage.

   (d) (5 points) Bill drops out of art school and gets an M.B.A. instead. He stops reading books and drinking coffee. Now he reads The Wall Street Journal and drinks bottled mineral water.

   His tastes have changed completely, and we do not know how he would respond to price and income changes. We need to observe how his consumption of the WSJ and bottled water change as his income changes.

2. (30 points) Assume a consumer's utility function is $U = \sqrt{q_1} + 2\sqrt{q_2}$ and her total income is \$90. The price of both good 1 and good 2 is \$1.

   (a) (5 points) What is the bundle that maximizes this consumer's utility? What is the consumer's utility level at that point?

We solve for the optimum bundle without replacing the price of good 1 until we obtain the demand for good 1 so we can calculate the optimum quantity with the new price later.

$$\mathcal{L} = \sqrt{q_1} + 2\sqrt{q_2} + \lambda(90 - p_1 q_1 - q_2)$$

$$\frac{d\mathcal{L}}{dq_1} = \frac{1}{2}q_1^{-1/2} - \lambda p_1 = 0 \qquad \rightarrow \frac{1}{2}q_1^{-1/2} = \lambda p_1 \tag{1}$$

$$\frac{d\mathcal{L}}{dq_2} = 2\frac{1}{2}q_2^{-1/2} - \lambda = 0 \qquad \rightarrow q_2^{-1/2} = \lambda \tag{2}$$

$$\frac{d\mathcal{L}}{d\lambda} = 90 - p_1 q_1 - q_2 = 0 \qquad \rightarrow 90 = p_1 q_1 + q_2 \tag{3}$$

We divide equations 1 and 2 to obtain:

$$\frac{\frac{1}{2}q_1^{-1/2}}{q_2^{-1/2}} = \frac{\lambda p_1}{\lambda}$$

$$\frac{1}{2}\left(\frac{q_1}{q_2}\right)^{-1/2} = p_1$$

$$\left(\frac{q_2}{q_1}\right)^{1/2} = 2p_1$$

$$\frac{q_2}{q_1} = (2p_1)^2$$

$$q_2 = 4p_1^2 q_1 \tag{4}$$

Replacing this result in the third first order condition (equation 3) we obtain the demand for good 1:

$$90 = p_1 q_1 + 4p_1^2 q_1 = (p_1 + 4p_1^2)q_1 \rightarrow q_1 = \frac{90}{(p_1 + 4p_1^2)} \tag{5}$$

Replacing this result back into equation 4, we obtain the demand for good 2 when the price of good 2 is $1 (which we replaced in the budget constraint at the beginning of the problem)

$$q_2 = 4p_1^2 \frac{90}{(p_1 + 4p_1^2)} \tag{6}$$

Equations 5 and 6 tell us that the utility maximizing bundle when $p_1 = 1$ is $q_2 = 72$ and $q_1 = 18$.

The consumer's utility derived from this bundle is $U = \sqrt{18} + 2\sqrt{72} \approx 21.21$

(b) (5 points) Suppose that the price of good 1 drops to $0.50. What is the new bundle that maximizes this consumer's utility? What is the consumer's utility at this point? [Note that in class we discussed an increase in price, while here we discuss a drop in price.]

We can now use equations 5 and 6 to obtain the optimum quantities of q_1 and q_2 for the new price of good 1.

$$q_1 = \frac{90}{0.5 + 4 \times (0.5)^2} = 60$$

$$q_2 = 4(0.5)^2 \frac{90}{0.5 + 4 \times (0.5)^2} = 60$$

The consumer's utility derived from this bundle is $U = \sqrt{60} + 2\sqrt{60} \approx 23.24$

(c) (10 points) What is the total effect on the quantity of good 1 of a drop in the price of good 1? Calculate the income and substitution effect of the drop in the price of good 1.

The total effect on the quantity of good 1 of a drop in its price is an increase from 18 to 60 units, i.e. 42 units.

In order to break this increase into income and substitution effects we need to calculate the optimum bundle that satisfies two conditions: it is on the old indifference curve but tangent to a budget constraint with the new prices.

Tangency condition: $MRS = \frac{p_1}{p_2}$

$$\frac{\frac{1}{2}q_1^{-1/2}}{2\frac{1}{2}q_2^{-1/2}} = \frac{0.5}{1}$$

$$\frac{1}{2}\left(\frac{q_1}{q_2}\right)^{-1/2} = 0.5$$

$$\left(\frac{q_2}{q_1}\right)^{1/2} = 1 \rightarrow \boxed{q_1 = q_2} \tag{7}$$

Remain at original utility level condition: in (a) we found that $U = 21.21$

$$U = \sqrt{q_1} + 2\sqrt{q_2} = 21.21$$

$$\text{Replacing equation 7: } \sqrt{q_1} + 2\sqrt{q_1} = 21.21$$

$$3\sqrt{q_1} = 21.21 \rightarrow \boxed{q_1 = 50}$$

The new bundle that is defined by a budget constraint with the new prices that is tangent to the initial indifference curve is $q_1 = q_2 = 50$. Because $p_1 = \$0.5$ and $p_2 = \$1$, total expenditure for this budget constraint is: $\$0.5 \times 50 + \$1 \times 50 = \$75$.

The substitution effect of the drop in p_1 is the change in q_1 between the original equilibrium and the equilibrium calculated here (where prices change but utility remains constant). The amount of q_1 increases from 18 to 50, so the substitution effect is 32 units.

The income effect of the drop in p_1 is the change in q_1 between the equilibrium calculated here and the final equilibrium (where prices are not changed and only income is changed).

The amount of q_1 increases from 50 to 60, so the income effect is 10 units.

(d) (10 points) How much income would compensate this consumer for this price change, i.e. how much income would this consumer need to give up in order to be equally well off under the new prices as she was before the prices changed?

This is the change in income between the original situation prior to the price change: $90 and the income necessary to leave the consumer at the original indifference curve but with the new prices. This is the bundle calculated in the previous question. Total expenditure for that bundle was $75. So the income that this consumer would have to give up in order to be equally well off under the new prices (i.e. on the same indifference curve as before the price change), is $15.

3. (30 points) A consumer's utility function is given by: $U(X,Y) = 10XY$. Currently, the prices of goods X and Y are $3 and $5, respectively, and the consumer's income is $150.

   a. (5 points) Find the MRS for this consumer for any given bundle (X,Y).

   The MRS is the ratio of marginal utilities:

$$MRS = \frac{MU_X}{MU_Y}$$
$$MRS = \frac{10Y}{10X}$$
$$MRS = \frac{Y}{X}$$

   b. (5 points) Find the optimal consumption bundle for this consumer and the level of utility the consumer derives from this bundle.

   The optimal consumption bundle is the which satisfies the budget constraint and where MRS =ratio of prices. Because we will look at a change in prices we derive the demands for the goods first and replace given prices at the end.

$$\frac{MU_X}{MU_Y} = \frac{p_X}{p_Y}$$
$$\frac{Y}{X} = \frac{p_X}{p_Y}$$
$$Y = \frac{p_X}{p_Y}X \tag{8}$$

We now replace this preliminary result in the budget constraint:

$$150 = p_X X + p_Y Y$$
$$150 = p_X X + p_Y \frac{p_X}{p_Y} X$$
$$150 = p_X X + p_X X$$
$$150 = 2p_X X \qquad \rightarrow \boxed{X = \frac{150}{2p_X}} \text{ Demand for good X} \tag{9}$$

We replace equation (9) back into equation (8)

$$Y = \frac{p_X}{p_Y}\frac{150}{2p_X} \rightarrow \boxed{Y = \frac{150}{2p_Y}} \text{ Demand for good Y}$$

So the optimal bundle given $p_X = 3$ and $p_Y = 5$ is $\boxed{X = 150/6 = 25}$ and $\boxed{Y = 150/10 = 15}$.

c. (10 points) Suppose the price of good X doubles. How much income is required so that the consumer is able to purchase the original consumption bundle, i.e. what is the cost-of-living adjusted income?

For the consumer to be able to purchase the same bundle ($X = 25, Y = 15$) when $p_X = 6$, income (I) would have to be:

$$I = 6 \times 25 + 5 \times 15 = 225$$

d. (10 points) Now that the price of good x has doubled, how much income is needed for the consumer to reach the original level of utility? Is this more or less that what you found in c., is the consumer overcompensated by being allowed enough income to purchase his original optimal bundle after the price change?

Here we do a similar procedure as in the previous exercise. We need to find the bundle that allows the consumer to achieve his initial level of utility but at the new prices. Two conditions need to be satisfied: the budget constraint with the new prices has to be tangent to an indifference curve and the level of utility has to be the same as for the original bundle.

Tangency condition:This is the same as what we did to derive equation (8), where prices have to be the new prices. So we are left with:

$$\boxed{Y = \frac{6}{5}X} \tag{10}$$

Original indifference curve: The level of utility at the original indifference curve can be found by replacing the original optimal bundle (in part b) in the utility function

$$U(X = 25, Y = 15) = 10 \times 25 \times 15 = 3750$$

We now know that the new bundle will have to also provide that level of utility.

$$\boxed{10XY = 3750} \tag{11}$$

We now combine equations 10 and 11:

$$\begin{aligned} 10XY &= 3750 \\ 10X\frac{6}{5}X &= 3750 \\ 10X^2 &= 3750 \rightarrow \boxed{X = 17.7} \end{aligned} \tag{12}$$

Replacing this back into equation 10

$$Y = \frac{6}{5}17.7 \rightarrow \boxed{Y = 21.2} \tag{13}$$

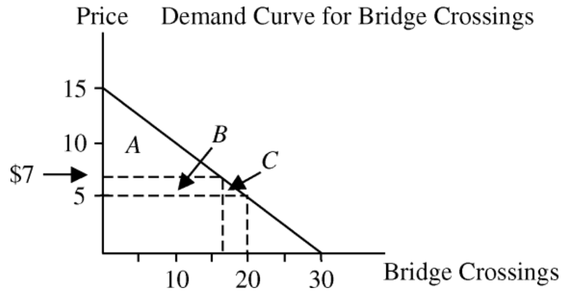
Now we only need to calculate the income required for the consumer to afford this bundle:

$$I = p_X X + P_Y Y = 6 \times 17.7 + 5 \times 21.2 = 212.1$$

So the income needed for the consumer to achieve the original level of utility at the new prices is \$212.1, which is less than the \$225 we found in part c. This means that if the consumer is provided enough income to purchase his original bundle after the price change (as in part c) he would be overcompensated, i.e. he would be able to achieve a higher level of utility than before the price change.

4. (20 points) Suppose you are in charge of a toll bridge that costs essentially nothing to operate. The demand for bridge crossings Q is given by $P = 15 - Q/2$, where P is the price of the toll and Q is the number of vehicles crossing the bridge.

   (a) (2.5 points) Draw the demand curve for bridge crossings as a function of the toll price.



(b) (2.5 points) How many people would cross the bridge if there were no toll?

At a price of zero, $0 = 15 - (1/2)Q \rightarrow Q = 30$. The quantity demanded would be 30.

(c) (5 points) What is the loss of consumer surplus associated with a bridge toll of $5?

- If $P = \$5 \rightarrow 5 = 15 - (1/2)Q \rightarrow Q = 20$. The consumer surplus is the area below the demand curve up to the market price. When there is no toll, the CS is the triangle determined by the origin, 15 on the y-axis and 30 on the x-axis. That area is: $15 \times 30/2 = 225$.
- After the price increase, the CS becomes areas A + B + C as labeled on the graph. That area is: $(15 - 5)(20)/2 = 100$.
- The loss in consumer surplus is $225 - 100 = 125$.

(d) (5 points) The toll-bridge operator is considering an increase in the toll to $7. At this higher price, how many people would cross the bridge? Would the toll-bridge revenue increase or decrease? What does your answer tell you about the elasticity of demand?

- If $P = \$7 \rightarrow 7 = 15 - (1/2)Q \rightarrow Q = 16$
- Revenue at $P = \$5$ is $R = 5 \times 20 = 100$, while revenue at $P = \$7$ is $R = 7 \times 16 = 112$. Revenue increases.
- Because revenue increases we know that demand must be inelastic: the increase in price outweighs the drop in quantity.
  We can also calculate elasticity = $\frac{\Delta Q}{\Delta P}\frac{P}{Q} = \frac{-4}{2}\frac{(7+5)/2}{(16+20)/2} = -0.66$, so demand is inelastic.

(e) (5 points) Find the lost consumer surplus associated with the increase in the price of the toll from $5 to $7.

- CS when $P = \$7$ is $(15 - 7)16/2 = 64$. We previously found that CS when $P = \$5$ was $100, so the loss in CS is $36.