ECONOMETRICS I
ECON 421/521
Midterm Exam, Fall 2014
October 8, 2014 – 11:45 - 13:00
Room: H-820
Instructor: Prosper Dovonon

NB: Read the questions carefully. Write clear and complete answers. Show your work and get credit. Partial answers get partial credit.

Question 1 [25 points]
Consider an equation to explain the salaries of CEOs in terms of annual firm sales:

$$\log(salary) = \beta_0 + \beta_1 \log(sales) + u.$$

(i) Give an interpretation of β_1 in this model.

(ii) The OLS estimate based on a sample of size $n = 209$ gives $\hat{\beta}_1 = 0.280$ with an $R^2 = 0.283$. Should this estimate be trusted as a fair value of what you expect from (i)? Make your case as clearly and convincing as possible.

Question 2 [75 points]
Consider the regression model

$$y_i = \beta x_i + u_i,$$

where $\{(x_i, y_i) : i = 1, \ldots, n\}$ is a random sample and $E(u_i|x_i) = 0$ and $Var(u_i|x_i) = \sigma^2$.

(i) Derive the OLS estimator $\hat{\beta}$ of β and show that $\hat{\beta} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$.

(ii) Show that $\hat{\beta}$ is unbiased and derive $Var(\hat{\beta}|X)$.
Now, consider the estimator of β constructed as $\tilde{\beta} = \frac{\bar{y}}{\bar{x}}$, where $\bar{y}$ and $\bar{x}$ are the sample mean of y_i and x_i.

(iii) Derive $E(\tilde{\beta})$. Is this estimator biased? Why?.

(iv) Derive $Var(\tilde{\beta}|X)$ and discuss which estimator is the most efficient between $\hat{\beta}$ and $\tilde{\beta}$.