Calc assignment #5

1. $\int_3^4 \frac{-3x^2+4x+4}{x}\,dx$

$= \int_3^4 \frac{-3x^2}{x}dx + \int_3^4 \frac{4x}{x}dx + \int_3^4 \frac{4}{x}dx$

$= -3\int_3^4 x\,dx + 4\int_3^4 1\,dx + 4\int_3^4 \frac{1}{x}\,dx$

$= -3\left(\frac{x^2}{2}\right)\Big|_3^4 + 4x\Big|_3^4 + 4\ln|x|\Big|_3^4$

$= -3\left(\frac{4^2}{2}\right) + 4(4) + 4\ln|4| - \left(-3\left(\frac{3^2}{2}\right) + 4(3) + 4\ln|3|\right)$

$= -24 + 16 + 5.545 + 13.5 - 12 - 4.3944$

$= -6.5 + 4\ln|4| - 4\ln|3|$

$= -6.5 + 4(\ln 4 - \ln 3)$

$= -6.5 + 4(\ln 4/3)$

2. $\int_3^5 4x^2\,dx + \int_5^7 3x+85\,dx$

$= 4\int_3^5 \frac{x^3}{3} + \int_5^7 \frac{3x^2}{2} + 85x$

$= 4\left(\frac{5^3}{3}\right) - 4\left(\frac{3^3}{3}\right) + \underbrace{\left(\frac{3(7)^2}{2} + 85(7)\right)}_{668.5} - \underbrace{\left(\frac{3(5)^2}{2} + 85(5)\right)}_{206}$

$= 4\left(\frac{5^3-3^3}{3}\right) + 206$

3. $\int_{-4}^4 |x|\,dx$

$= \frac{4\sqrt{4^2}}{2} - \frac{-4\sqrt{-4^2}}{2}$

$= 2 \cdot -$

4. $\int_2^x 3t+3\,dx$

$= \int_2^x \frac{3t^2}{2} + 3t$

$= \frac{3x^2}{2} + 3x - 12$

5. $\int_u^3 \cos x$

$= \sin 3 - \sin u$

$\frac{d}{dx} \cancel{\sin 3} - \sin u$

$\hookrightarrow C$

$= \cos 3 - \cos u$

$\int_{e^{3t}}^3 -\cos(e^{3t})$

$= \sin 3 - \sin(e^{3t})$

$= \frac{d}{dx} \sin 3 - \sin(e^{3t})$

$= \cancel{-\cos(e^{3t})(e^{3t})3}$

$= -3e^{3x}\cos(e^{3x})$

6. $\int_0^6 4x(u)^{1/4}$

$= 4u^{1/4} \int_0^6 x\,dx$

$= 4u^{1/4} \int_0^6 \frac{x^2}{2}$

$= \int_0^6 2u^{1/4} x^2$

$u = 36 - x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$-\frac{1}{2} du = x\,dx$

$\int_0^6 4xu^{1/4}$

$-\frac{4}{2}\int_0^6 (u)^{1/4}$

$-\frac{4}{2}\frac{u^{5/4}}{5/4}$

$= -2 \cdot \frac{4u^{5/4}}{5}$

$= -\frac{8u^{5/4}}{5}$

$= -\frac{8(36-x^2)^{5/4}}{5}$

$= \frac{-8(36-(6)^2)^{5/4}}{5} - \frac{-8(36-(0)^2)^{5/4}}{5}$

7. $\int \frac{e^x}{\cos^2(e^x)}dx = \int \frac{u}{\cos^2 u}\frac{1}{u}du$

$e^x = u$

$\frac{du}{dx} = u$

$du = u\,dx$

$dx = u\,du$

$= \int \frac{1}{\cos^2(u)}du$

$= \int \sec^2(u)\,du$

$= \tan(u)$

$= \tan(e^x)$

8. $\int \frac{e^u}{u}\,dx$ $\quad u=\sqrt{5x+3}$

$\frac{du}{dx}=\frac{1}{2\sqrt{5x+3}}(5)$

$=\int \frac{e^u}{u}\left(\frac{2\sqrt{u}}{5}\right)du \quad \frac{du}{dx}=\frac{5}{2u}$

$=\int \frac{e^u}{\cancel{x}}\cdot\frac{2\cancel{x}}{5}\,du \quad du\frac{2u}{5}=dx$

$=\int \frac{2e^u}{5}\,du$

$=\frac{2}{5}\int e^u$

$=\frac{2}{5}e^{\sqrt{5x+3}}$

9. $\int_e^{e^2} \frac{1}{x(\ln x)^3}\,dx \quad u=\ln x \quad \frac{du}{dx}=\frac{1}{x} \quad du=\frac{dx}{x} \quad x\,du=dx$

$=\int_e^{e^2} \frac{1}{\cancel{x}u^3}\cancel{x}\,du$

$=\int_e^{e^2} \frac{1}{u^3}$

$=\int_e^{e^2} u^{-3}$

$=\int_e^{e^2} \frac{u^{-2}}{-2}$

$=\int_e^{e^2} -\frac{1}{2u^2}$

$=-\frac{1}{2(\ln e^2)^2}-\frac{1}{2(\ln e)^2}$

$=-\frac{1}{8}+\frac{1}{2}$

$=3/8$

10. $\int \frac{1+6t}{1+t^2}\,dt$

$=\int \frac{1}{1+t^2}+\int\frac{6t}{1+t^2}$

$=\tan^{-1}x+6\int\frac{t}{1+t^2}\,dt \quad u=1+t^2 \quad \frac{du}{dt}=2t$

$=\tan^{-1}x+6\int\frac{\cancel{t}}{u}\frac{1}{2\cancel{t}}\,du \quad du=2t\,dt \quad \frac{1}{2t}du=dt$

$=\tan^{-1}t+3\int\frac{1}{u}$

$=\tan^{-1}t+3\ln|u|$

$=\tan^{-1}t+3\ln|1+t^2|$

Maple doesn't accept $|1+t^2|$ need $(1+t^2)$