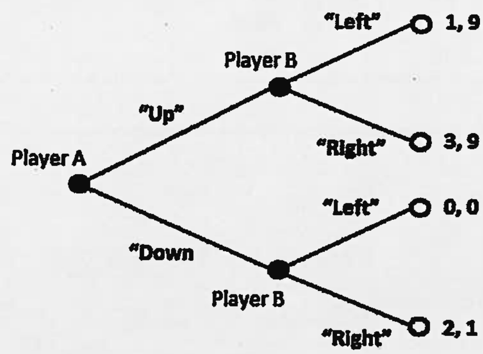
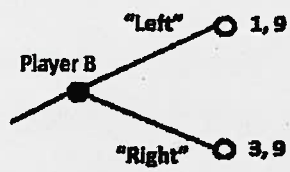
ECON302

Tutorial 8 & 9

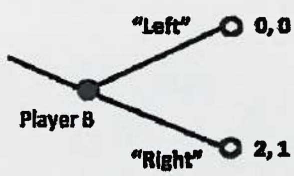
1. Consider the following twist in the sequential game presented in class:



(a) There are two subgames in this game (not including the big game). The subgames are:



(subgame 1)



(subgame 2)

Player A decides only ones (one decision node). Therefore, a complete strategy of player A should report one action. Player B has two decision nodes so a complete strategy of player B should report two actions, one for every sub-game.

(b) There is no BI outcome and no SPNE in this game. The reason is that in subgame 1 player B is indifferent between his two actions as they both pay him 9. However, this makes a big difference for player A since if it is known that player B is choosing "Left" in subgame 1, player A would prefer to go "Up" and receive 3 instead of going "Down" and, given that player B os choosing "Right" in the second subgame, receive 2. On the other hand, if it is known that player B is choosing "Right" in subgame 1, player A would prefer to go "Down" and, given that player B os choosing "Right" in the second subgame, receive 2 instead of going "Up" to receive 1! The problem is that we do not know if player B is a "nice guy" or a "bad guy"!

2. Consider a general equilibrium model where there are two persons, A and B, and two goods, 1 and 2 (good 2 is the "numeraire," *i.e.*, its price can be normalized to unit once you find the equilibrium price ratio). Person A has 9 units of good 1 and 1 unit of good 2 while person B has 3 units of good 1 and 8 units of good 2. Person A's utility is represented by $U_A = \left(x_1^A\right)^{1/3}\left(x_2^A\right)^{2/3}$ and person B's utility is given by $U_B = \left(x_1^B\right)^{2/3}\left(x_2^B\right)^{1/3}$. The corresponding Marshallian demands are given by $x_1^A = (1/3)\left(m^A/p_1\right)$, $x_2^A = (2/3)\left(m^A/p_2\right)$, $x_1^B = (2/3)\left(m^B/p_1\right)$, and $x_2^B = (1/3)\left(m^B/p_2\right)$, where m^j is the income of person $j = A, B$.

(a) The total endowments are

$$w_1 = w_1^A + w_1^B \Rightarrow w_1 = 9 + 3 = 12$$

and

$$w_2 = w_2^A + w_2^B \Rightarrow w_2 = 1 + 8 = 9$$

First we examine feasibility:

- the first allocation is NOT feasible since $7 + 5 = 12$ and $7 + 4 = 11 > 9$
- the second allocation is feasible since $5 + 7 = 12$ but $6 + 3 = 9$
- the third allocation is feasible BUT it does not exhaust the endowment since $6 + 5 = 11 < 12$ and $7 + 2 = 9$.

(b) The marginal rates of substitution are

$$MRS^A = \frac{\frac{\partial U_A}{\partial x_1^A}}{\frac{\partial U_A}{\partial x_2^A}} = \frac{\frac{1}{3}\left(x_1^A\right)^{-2/3}\left(x_2^A\right)^{2/3}}{\frac{2}{3}\left(x_1^A\right)^{1/3}\left(x_2^A\right)^{-1/3}} = \frac{x_2^A}{2x_1^A}$$

and

$$MRS^B = \frac{\frac{\partial U_B}{\partial x_1^B}}{\frac{\partial U_B}{\partial x_2^B}} = \frac{\frac{2}{3}\left(x_1^B\right)^{-1/3}\left(x_2^B\right)^{1/3}}{\frac{1}{3}\left(x_1^B\right)^{2/3}\left(x_2^B\right)^{-2/3}} = \frac{2x_2^B}{x_1^B}$$

Checking for Pareto efficiency:

- The first allocation cannot be Pareto Optimal since it is not feasible!
- For the second allocation we get we get

$$MRS^A = \frac{6}{2(5)} = 0.6 \neq MRS^B = \frac{2(3)}{(7)} = 0.86$$

so it is NOT Pareto Optimal.

- The third allocation cannot be Pareto Optimal since it does not exhaust the endowment!

(c) Based on their endowments, the incomes of the two consumers are

$$m^A = 9p_1 + 1p_2,$$

and

$$m^B = 3p_1 + 8p_2,$$

correspondingly. Given these incomes and their Marshallian demands, the gross demands for each one of the consumers are

$$x_1^A = \frac{1}{3}\frac{9p_1 + 1p_2}{p_1} \Rightarrow x_1^A = \frac{9p_1 + p_2}{3p_1},$$

$$x_2^A = \frac{2}{3}\frac{9p_1 + 1p_2}{p_2} \Rightarrow x_2^A = \frac{18p_1 + 2p_2}{3p_2},$$

$$x_1^B = \frac{2}{3}\frac{3p_1 + 8p_2}{p_1} \Rightarrow x_1^B = \frac{6p_1 + 16p_2}{3p_1},$$

and

$$x_2^B = \frac{1}{3}\frac{3p_1 + 8p_2}{p_2} \Rightarrow x_2^B = \frac{3p_1 + 8p_2}{3p_2},$$

correspondingly.

(d) Their excess demand functions are given by

$$e_1^A = x_1^A - w_1^A = \frac{9p_1 + p_2}{3p_1} - 9 \Rightarrow e_1^A = \frac{p_2 - 18p_1}{3p_1},$$

$$e_2^A = x_2^A - w_2^A = \frac{18p_1 + 2p_2}{3p_2} - 1 \Rightarrow e_2^A = \frac{18p_1 - p_2}{3p_2},$$

$$e_1^B = x_1^B - w_1^B = \frac{6p_1 + 16p_2}{3p_1} - 3 \Rightarrow e_1^B = \frac{16p_2 - 3p_1}{3p_1},$$

and

$$e_2^B = x_2^B - w_2^B = \frac{3p_1 + 8p_2}{3p_2} - 8 \Rightarrow e_1^B = \frac{3p_1 - 16p_2}{3p_2},$$

correspondingly. The corresponding aggregate excess demands are

$$z_1 = e_1^A + e_1^B = \frac{p_2 - 18p_x}{3p_1} + \frac{16p_2 - 3p_x}{3p_1} \Rightarrow z_1 = \frac{17p_2 - 21p_1}{3p_1},$$

and

$$z_2 = e_2^A + e_2^B = \frac{18p_1 - p_2}{3p_2} + \frac{3p_1 - 16p_2}{3p_2} \Rightarrow z_2 = \frac{21p_1 - 17p_2}{3p_2}.$$

(e) According to Walras law the aggregate value of all aggregate excess demands is identically equal to zero. That is,

$$p_1 z_1 + p_2 z_2 = 0.$$

This can be verified for this problem as

$$p_1 z_1 + p_2 z_2 = p_1 \frac{17p_2 - 21p_1}{3p_1} + p_2 \frac{21p_1 - 17p_2}{3p_2} = 0.$$

(f) Due to Walras law it suffices that one of the two markets clears. Therefore, the Walrasian equilibrium can be found by

$$z_1 = 0 \Rightarrow \frac{17p_2 - 21p_1}{3p_1} = 0 \Rightarrow 17p_2 - 21p_1 = 0 \Rightarrow \frac{p_1^*}{p_2^*} = \frac{17}{21}.$$

(g) Since good y is the numeraire we can set $p_2^* = 1$ and, therefore, $p_1^* = 17/21$. Plugging these prices in the individual gross demands from part *(i)* we get

$$x_1^A = \frac{9\,(17/21) + 1}{3\,(17/21)} = \frac{58}{17},$$

$$x_2^A = \frac{18\,(17/21) + 2\,(1)}{3\,(1)} = \frac{116}{21},$$

$$x_1^B = \frac{6\,(17/21) + 16\,(1)}{3\,(17/21)} = \frac{146}{17},$$

and

$$x_2^B = \frac{3\,(17/21) + 8\,(1)}{3\,(1)} = \frac{73}{21}.$$