Econ 302

Tutorial 6

1. Let player A assign probability r to his action "Straight" (so, probability $1-r$ is assigned to his action "Turn") and let player B assign probability c to his action "Straight" (so, probability $1-c$ is assigned to his action "Turn"). In that case the expected payoff of player A is

$$EU_A = (-2)\,rc + (1)\,r\,(1-c) + (-1)\,(1-r)\,c + (0)\,(1-r)\,(1-c) \Rightarrow EU_A = r - c - 2cr$$

and the derivative with respect to his choice (r) is

$$\frac{\partial EU_A}{\partial r} = 1 - 2c$$

We distinguish the following cases (best response of player A):

- if $1-2c > 0 \Rightarrow c < 1/2$ then it is best for player A to make r as big as possible, i.e., $r^* = 1$
- if $1-2c < 0 \Rightarrow c > 1/2$ then it is best for player A to make r as small as possible, i.e., $r^* = 0$
- if $1-2c = 0 \Rightarrow c = 1/2$ then player A is indifferent between his choices over r, i.e., $r^* \in [0,1]$

The expected payoff of player B is

$$EU_B = (-2)\,rc + (-1)\,r\,(1-c) + (1)\,(1-r)\,c + (0)\,(1-r)\,(1-c) \Rightarrow EU_A = c - r - 2cr$$

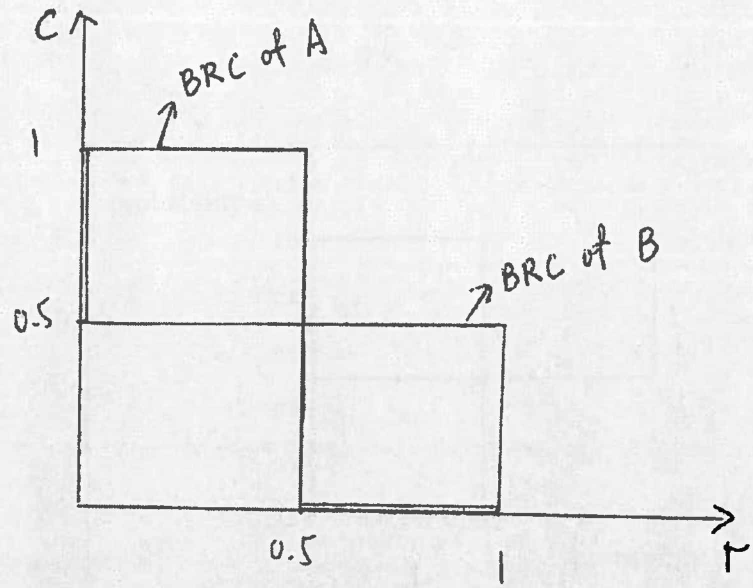
and the derivative with respect to his choice (c) is

$$\frac{\partial EU_B}{\partial c} = 1 - 2r$$

We distinguish the following cases (best response of player B):

- if $1-2r > 0 \Rightarrow r < 1/2$ then it is best for player B to make c as big as possible, i.e., $c^* = 1$
- if $1-2r < 0 \Rightarrow r > 1/2$ then it is best for player B to make c as small as possible, i.e., $c^* = 0$
- if $1-2r = 0 \Rightarrow r = 1/2$ then player B is indifferent between his choices over c, i.e., $c^* \in [0,1]$

Graphically, we get 0.5



and the solution is

$$NEMS = \left\{ \frac{1}{2}S\frac{1}{2}T, \frac{1}{2}S\frac{1}{2}T \right\}$$

2. The payoff matrix is

		Player B		
		Left	Center	Right
Player A	Up	5, 5	6, 1	-2, 4
	Down	6, 1	1, 1	-1, 2

(b) No, there are no dominant strategies. Player A would prefer to play "Down" when player B plays "Left" but he will play "Up" when player B chooses "Center." That shows that player A does change his strategy depending on the choice of player B. Similarly, player B would prefer to play "Left" when player A plays "Up" but he will play "Right" when player A chooses "Down." That shows that player B does change his strategy depending on the choice of player A.

(c) Nash Equilibrium is

$$NE_1 = ("Down", "Right")$$

with payoffs $(-\$1, \$2)$. This is a Nash Equilibrium since no player has an incentive to unilaterally deviate. More specifically, given that player B chooses "Right," player A does not want to choose "Up" instead of "Down" because in this case he will lose \$2 instead of loosing \$1. Similarly, given that player A chooses "Down," player B does not want to choose "Left" or "Center" instead of "Right" because in either case he will get \$1 instead of \$2. Therefore, no player wants to change his decision and the suggested set of strategies is indeed a Nash Equilibrium.

(d) First note that player B NEVER chooses "Center." Therefore, we can eliminate the middle column in the payoff matrix of the original game. Thus we get

		Player B	
		c	(1-c)
		Left	Right
Player A r	Up	5, 5	-2, 4
Player A (1-r)	Down	6, 1	-1, 2

where r and c are the probabilities players A and B assign to their actions "Up" and "Left" correspondingly. The expected payoffs are then

$$EU_A = rc(5) + r(1-c)(-2) + (1-r)c(6) + (1-r)(1-c)(-1) \Rightarrow EU_A = 7c - r - 1$$

and

$$EU_B = rc(5) + r(1-c)(4) + (1-r)c(1) + (1-r)(1-c)(2) \Rightarrow EU_B = 2r - c + 2cr + 2.$$

When player A is trying to maximize his expected payoff (by optimally choosing r) we observe that

$$\frac{\partial EU_A}{\partial r} = -1 < 0.$$

This implies that he should make r as small as possible. Since $r \in [0,1]$ we get $r^* = 0$, i.e., he will play "Down" with probability 100%. For player B's maximization we observe that

$$\frac{\partial EU_B}{\partial c} = -1 + 2r.$$

Therefore, there are three cases:

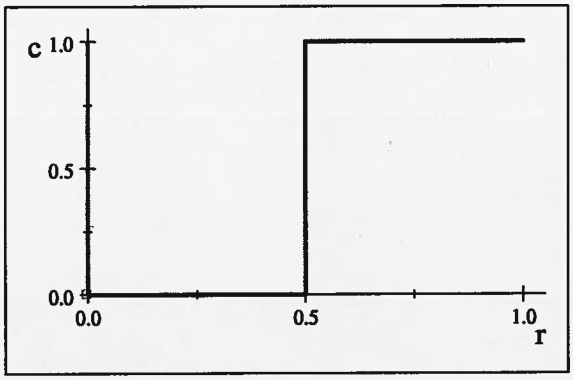
$$\begin{aligned}
&\text{(i)} & -1+2r &> 0 \Rightarrow r > 1/2. \text{ In this case } \frac{\partial EU_B}{\partial c} > 0 \text{ and } c^* = 1, \\
&\text{(ii)} & -1+2r &< 0 \Rightarrow r < 1/2. \text{ In this case } \frac{\partial EU_B}{\partial c} < 0 \text{ and } c^* = 0, \text{ and} \\
&\text{(iii)} & -1+2r &= 0 \Rightarrow r = 1/2. \text{ In this case } \frac{\partial EU_B}{\partial c} = 0 \text{ and } c^* \in [0,1].
\end{aligned}$$

The MSNE in this case is easy to be identified as player A always chooses $r^* = 0$. Given that $r^* < 1/2$, player B's best response is to choose $c^* = 0$. Therefore,

$$MSNE = (0 \times \text{"}Up\text{"} \text{ and } 1 \times \text{"}Down\text{"}, \; 0 \times \text{"}Left\text{"} \text{ and } 1 \times \text{"}Right\text{"})$$

with payoffs $(-1, 2)$.

(e) Graphically the best responses are presented in the graph below. Note that there is only one NE in pure and/or mixed strategies.



(f) The resulting NE in pure strategies is the worst case in this game since the sum of payoffs $(-1+2=1)$ is the smallest among all the cells of the original game. Unfortunately, when the players are randomizing their actions the same result is sustained. The socially desirable outcome, i.e., cell ("Up," "Left") with payoffs $(5,5)$ cannot be sustained with the current format of the game.

3. Consider a game where player A decides between his options, namely "Up", "Middle" and "Down" and Player B has to choose between "Left" or "Right." The payoff matrix is given by

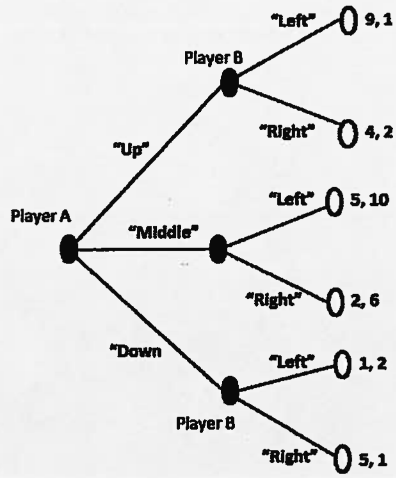
		Player B	
		Left	Right
Player A	Up	9, 1	4, 2
	Middle	5, 10	2, 6
	Down	1, 2	5, 1

(a) When Player B plays "Left" is best for player A to play "Up." When Player B plays "Right" is best for player A to play "Down." No dominant strategy.

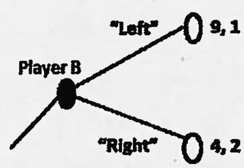
(b) When Player A plays "Up" is best for player B to play "Right." When Player A plays "Middle" is best for player B to play "Left." When Player A plays "Down" is best for player B to play "Left."

(c) There are no Nash Equilibria in pure strategies in this game. For example, ("Up," "Left") is not an Nash Equilibrium because at least one player would like to change his/her decision! In this case, player B will deviate from this cell as, given that player A plays "Up," player B will find it profitable to choose "Right" instead of "Left."

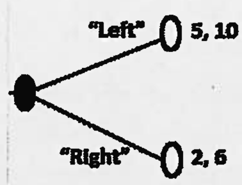
(d) The extensive form representation of this game is



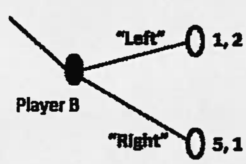
(e) There are three subgames in this game (not including the big game). The subgames are:



(subgame 1)



(subgame 2)



(subgame 3)

Player A decides only ones (one decision node). Therefore, a complete strategy of player A should report one action. Player B has three decision nodes so a complete strategy of player B should report three actions, one for every sub-game.

(f) The Backwards Induction (BI) of this game is (Middle, Left). The Subgame Perfect Nash Equilibrium (SPNE) of this game is "Middle" for player A and "Right when Up," "Left when Middle," and "Left when Down" for player B.