

Econ 302
Suggested answers for tutorial 5

1. (a) The follower's maximization problem is

$$\max_{y_2} \Pi_2 = py_2 - c_2(y_2) = (10 - 2y_1 - 2y_2)y_2 - 2y_2^2$$

and the first order conditions yield

$$\frac{\partial \Pi}{\partial y_2} = 0 \Rightarrow 10 - 8y_2 - 2y_1 = 0 \Rightarrow y_2 = \frac{10 - 2y_1}{8}$$

- (b) The leader's maximization problem is

$$\max_{y_1} \{ \Pi_1 = py_1 - c_1(y_1) \} \Rightarrow$$

$$\max_{y_1} \{ \Pi_1 = (10 - 2y_1 - 2y_2)y_1 - y_1^2 \} \Rightarrow$$

$$\max_{y_1} \left\{ \Pi_1 = (10 - 2y_1 - 2 \frac{10 - 2y_1}{8})y_1 - y_1^2 \right\}$$

and the first order conditions yield

$$\frac{\partial \Pi_1}{\partial y_1} = 0 \Rightarrow \frac{15}{2} - 5y_1 = 0 \Rightarrow y_1^* = 1.5$$

- (c) Follower's reaction function evaluated at the leader's optimal choice found in part (b) yields

$$y_2^* = \frac{10 - 2y_1^*}{8} \Rightarrow y_2^* = \frac{10 - 2(1.5)}{8} \Rightarrow y_2^* = 0.875$$

- (d) Given the optimal quantities found in parts (b) and (c) we get

$$P^* = 10 - 2y_1^* - 2y_2^* \Rightarrow P^* = 10 - 2(1.5) - 2(0.875) \Rightarrow P^* = \$5.25$$

Furthermore, the corresponding profits will be

$$\Pi_1^* = p^* y_1^* - c_1(y_1^*) \Rightarrow \Pi_1^* = (5.25)1.5 - 1.5^2 \Rightarrow \Pi_1^* = \$5.625,$$

and

$$\Pi_2^* = p^* y_2^* - c_2(y_2^*) \Rightarrow \Pi_2^* = (5.25)0.875 - 2(0.875)^2 \Rightarrow \Pi_2^* = \$3.0625.$$

2. (a) The follower's maximization problem is

$$\max_{y_2} \Pi_2 = py_2 - c_2(y_2) = (10 - 2y_1 - 2y_2)y_2 - (8y_2 + 2y_2^2)$$

and the first order conditions yield

$$\frac{\partial \Pi}{\partial y_2} = 0 \Rightarrow 2 - 8y_2 - 2y_1 = 0 \Rightarrow y_2 = \frac{1 - y_1}{4}$$

(b) The leader's maximization problem is

$$\max_{y_1} \{ \Pi_1 = py_1 - c_1(y_1) \} \Rightarrow$$

$$\max_{y_1} \{ \Pi_1 = (10 - 2y_1 - 2y_2)y_1 - y_1^2 \} \Rightarrow$$

$$\max_{y_1} \left\{ \Pi_1 = (10 - 2y_1 - 2\frac{1-y_1}{4})y_1 - y_1^2 \right\}$$

and the first order conditions yield

$$\frac{\partial \Pi_1}{\partial y_1} = 0 \Rightarrow \frac{19}{2} - 5y_1 = 0 \Rightarrow y_1^* = 1.9$$

(c) What will be the follower's optimal quantity? Follower's reaction function evaluated at the leader's optimal choice found in part (b) yields

$$y_2^* = \frac{1 - y_1^*}{4} \Rightarrow y_2^* = \frac{1 - 1.9}{8} \Rightarrow y_2^* = -0.9.$$

But this answer obviously makes no sense. It means that the leader is "strong" enough to produce a quantity that will keep the follower out of the market, i.e.,

$$y_2^* = 0.$$

But is then $y_1^* = 1.9$ optimal for firm 1? If firm 2 is out of the market, firm 1 has to take this into account and therefore, firm 1's profit maximization is updated to

$$\max_{y_1} \{ \Pi_1 = py_1 - c_1(y_1) \} \Rightarrow$$

$$\max_{y_1} \{ \Pi_1 = (10 - 2y_1)y_1 - y_1^2 \}$$

and the first order conditions yield

$$\frac{\partial \Pi_1}{\partial y_1} = 0 \Rightarrow 10 - 6y_1 = 0 \Rightarrow y_1^* = 1.67.$$

Note that for this quantity, firm 2 STILL REMAINS OUT OF THE MARKET as $y_2^* = \frac{1 - y_1^*}{4} \Rightarrow y_2^* = \frac{1 - 1.67}{8} \Rightarrow y_2^* = -0.67$.

(d) Given the optimal quantities found we get

$$P^* = 10 - 2y_1^* - 2y_2^* \Rightarrow P^* = 10 - 2(1.67) - 2(0) \Rightarrow P^* = \$6.67$$

Furthermore, the corresponding profits will be

$$\Pi_1^* = p^* y_1^* - c_1(y_1^*) \Rightarrow \Pi_1^* = (6.67) 1.67 - 1.67^2 \Rightarrow \Pi_1^* = \$8.35,$$

and

$$\Pi_2^* = 0.$$

3. (a) An equilibrium pricing scenario (NOT UNIQUE) is given by

$$\begin{cases} p_1^* = \$1.18 \\ p_2^* = \$1.19 \end{cases}$$

According to these prices, firm 1 gets the entire market, that is

$$D_1(p_1, p_2) = D(p) = 800 - 50(1.18) \Rightarrow y_1 = 741$$

Moreover, firm 2 gets no market share, i.e.,

$$y_2 = 0$$

The corresponding profits are

$$\Pi_1 = p_1 y_1 - 1.17 y_1 = 1.18(741) - 1.17(741) = \$7.41$$

and

$$\Pi_2 = p_2 y_2 - 1.19 y_2 = 1.19(0) - 1.19(0) = \$0$$

This suggested pricing scenario is an equilibrium because no firm has incentive to charge different price than the one suggested. More specifically,

- Firm 1 does not want to charge more than $p_1 = \$1.18$. If firm 1 charges \$1.20 or more it will be getting no customers (since $p_2 = 1.19$ will be the lowest price) making a profit of \$0. Moreover, by charging a price of \$1.19, firm 1 will be sharing the market with firm 2. The total market quantity will be

$$Y = 800 - 50(1.19) = 740.5$$

and firm 1's quantity is $y_1 = 740.5/2 = 370.25$. The corresponding profits will be

$$\Pi_1 = p_1 y_1 - 1.17 y_1 = 1.19(370.25) - 1.17(370.25) = \$7.405$$

In any case, firm 1 will be making less money by increasing its price above \$1.18. Firm 1 does not want to charge less than \$1.18 either since obviously its profits will decrease.

- Firm 2 does not want to charge more than $p_2 = \$1.19$. If firm 2 charges \$1.20 or more it will still be getting no customers (since $p_1 = 1.18$ is the lowest price) making a profit of \$0. Firm 2 does not want to charge less than \$1.19 either since it will be making a loss.

(b) Given the pricing scenario you proposed in (a), find the quantities that each firm is selling, the total market quantity, and the corresponding profits. An equilibrium pricing scenario (UNIQUE) is given by

$$\begin{cases} p_1^* = \$1.20 \\ p_2^* = \$1.20 \end{cases}$$

According to these prices the two firms equally share the market, i.e.,

$$y_1 = y_2 = \frac{1}{2} D(p) = \frac{1}{2} [800 - 50(1.20)] \Rightarrow y_1 = y_2 = 370$$

The corresponding profits are

$$\Pi_1 = p_1 y_1 - 1.17 y_1 = 1.20 (370) - 1.17 (370) = \$11.10,$$

and

$$\Pi_2 = p_2 y_2 - 1.19 y_2 = 1.20 (370) - 1.19 (370) = \$3.70$$

This suggested pricing scenario is an equilibrium because no firm has incentive to charge different price than the one suggested. More specifically,

- Firm 1 does not want to charge more than $p_1 = \$1.20$. If firm 1 charges \$1.25 (remember: there are no pennies in this economy!) or more it will be getting no customers (since $p_2 = 1.20$ will be the lowest price) making a profit of \$0. Moreover, by charging a price of \$1.15, firm 1 will get all the customers but it will make losses (as the price will be less than the marginal cost). Therefore, Firm 1 does not want to charge either more or less than \$1.20 since obviously its profits will decrease.
- A similar argument holds for firm 2!