Econ 302
Suggested answers for tutorial 4

1. (a) Firm 1's maximization problem is

$$\max_{y_1} \Pi_1 = py_1 - c_1(y_1) = (10 - 2y_1 - 2y_2)y_1 - y_1^2$$

and the first order conditions yield

$$\frac{\partial \Pi}{\partial y_1} = 0 \Rightarrow 10 - 6y_1 - 2y_2 = 0 \Rightarrow y_1 = \frac{10 - 2y_2}{6}$$

(b) Firm 2's maximization problem is

$$\max_{y_2} \Pi_2 = py_2 - c_2(y_2) = (10 - 2y_1 - 2y_2)y_2 - 2y_2^2$$

and the first order conditions yield

$$\frac{\partial \Pi}{\partial y_2} = 0 \Rightarrow 10 - 8y_2 - 2y_1 = 0 \Rightarrow y_2 = \frac{10 - 2y_1}{8}$$

(c) The reaction functions derived in parts (a) and (b) form a 2×2 system of equations. Solving that system (using the substitution method) yields

$$\left.\begin{array}{l} y_1 = \dfrac{10 - 2y_2}{6} \\ y_2 = \dfrac{10 - 2y_1}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y_1 = \dfrac{10 - 2\left(\dfrac{10 - 2y_1}{8}\right)}{6} \\ y_2 = \dfrac{10 - 2y_1}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 6y_1 = 10 - \dfrac{10 - 2y_1}{4} \\ y_2 = \dfrac{10 - 2y_1}{8} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} 6y_1 = 10 - 2.5 + 0.5y_1 \\ y_2 = \dfrac{10 - 2y_1}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 5.5y_1 = 7.5 \\ y_2 = \dfrac{10 - 2y_1}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y_1^* = 1.36 \\ y_2 = \dfrac{10 - 2y_1}{8} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} y_1^* = 1.36 \\ y_2 = \dfrac{10 - 2(1.36)}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y_1^* = 1.36 \\ y_2^* = 0.91 \end{array}\right.$$

(d) Given the optimal quantities found in part (c) we get

$$P^* = 10 - 2y_1^* - 2y_2^* \Rightarrow P^* = 10 - 2(1.36) - 2(0.91) \Rightarrow P^* = \$5.46$$

Furthermore, the corresponding profits will be

$$\Pi_1^* = p^* y_1^* - c_1(y_1^*) \Rightarrow \Pi_1^* = (5.46)\,1.36 - 1.36^2 \Rightarrow \Pi_1^* = \$5.576,$$

and

$$\Pi_2^* = p^* y_2^* - c_2(y_2^*) \Rightarrow \Pi_2^* = (5.46)\,0.91 - 2(0.91)^2 \Rightarrow \Pi_2^* = \$3.3124.$$

2. (a) The cartel maximizes the joint profits, i.e.,

$$\max_{y_1, y_2} \Pi_{1+2} = py_1 - c_1(y_1) + py_2 - c_2(y_2) = (10 - 2y_1 - 2y_2)(y_1 + y_2) - y_1^2 - 2y_2^2$$

(b) The first order conditions are

$$\frac{\partial \Pi_{1+2}}{\partial y_1} = 0 \Rightarrow 10 - 6y_1 - 4y_2 = 0 \Rightarrow 6y_1 + 4y_2 = 10,$$

and

$$\frac{\partial \Pi_{1+2}}{\partial y_2} = 0 \Rightarrow 10 - 8y_2 - 4y_1 = 0 \Rightarrow 4y_1 + 8y_2 = 10.$$

Solving these two as a system yields

$$\left.\begin{array}{l} 6y_1 + 4y_2 = 10 \\ 4y_1 + 8y_2 = 10 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 6y_1 + 4y_2 = 10 \\ y_2 = \dfrac{10 - 4y_1}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 6y_1 + 4\left(\dfrac{10 - 4y_1}{8}\right) = 10 \\ y_2 = \dfrac{10 - 4y_1}{8} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} 6y_1 + 5 - 2y_1 = 10 \\ y_2 = \dfrac{10 - 4y_1}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} 4y_1 = 5 \\ y_2 = \dfrac{10 - 4y_1}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y_1^* = 1.25 \\ y_2 = \dfrac{10 - 4y_1}{8} \end{array}\right\} \Rightarrow$$

$$\Rightarrow \left\{\begin{array}{l} y_1^* = 1.25 \\ y_2 = \dfrac{10 - 4(1.25)}{8} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} y_1^* = 1.25 \\ y_2^* = 0.625 \end{array}\right.$$

(c) Given our answer in part (a), the total quantity produced under the cartel agreement will be

$$Y = y_1^* + y_2^* = 1.25 + 0.625 = 1.875,$$

and the corresponding price will be

$$p^* = 10 - 2Y^* = 10 - 2(1.875) \Rightarrow p^* = \$625.$$

The industry profit for the cartel will be

$$\Pi_{1+2}^* = p^* y_1^* - c_1(y_1^*) + p^* y_2^* - c_2(y_2^*) = 6.25(1.25) - (1.25)^2 + 6.25(0.625) - 2(0.625)^2 = \$9.375.$$

Note that these profits will be distributed according to the bargaining power each one of the firms have when negotiating. The only requirement is that each firm will be receiving at least as much as under a Cournot solution. If a firm cannot guarantee the Cournot profit, it will have no incentive to participate in a cartel agreement.

2. (a) The cartel maximizes the joint profits, i.e.,

$$\max_{y_1,y_2} \Pi_{1+2} = py_1 - c_1(y_1) + py_2 - c_2(y_2) = (10 - 2y_1 - 2y_2)(y_1 + y_2) - y_1 - 2y_2$$

(b) The first order conditions are

$$\frac{\partial \Pi_{1+2}}{\partial y_1} = 0 \Rightarrow 9 - 4y_2 - 4y_1 = 0 \Rightarrow 4y_1 + 4y_2 = 9,$$

and

$$\frac{\partial \Pi_{1+2}}{\partial y_2} = 0 \Rightarrow 8 - 4y_2 - 4y_1 = 0 \Rightarrow 4y_1 + 4y_2 = 8.$$

Solving these two as a system yields NO SOLUTION (impossible system). The reason is because the cartel would adopt a "corner" solution, namely all the units that the cartel will decide to produce they will be produced in the facilities of firm 1 BECAUSE $MC_1 > MC_2$ for any level of output. Therefore, the profit maximization problem for the cartel is updated to

$$\max_{y_1,y_2=0} \Pi_{1+2} = py_1 - c_1(y_1) = (10 - 2y_1)y_1 - y_1$$

with FOC being

$$\frac{\partial \Pi_{1+2}}{\partial y_1} = 0 \Rightarrow 10 - 4y_1 - 1 = 0 \Rightarrow 4y_1 = 9 \Rightarrow y_1 = 2.25.$$

(c) Given our answer in part (a), the total quantity produced under the cartel agreement will be

$$Y = y_1^* + y_2^* = 2.25 + 0 = 2.25,$$

and the corresponding price will be

$$p^* = 10 - 2Y^* = 10 - 2(2.25) \Rightarrow p^* = \$5.50.$$

The industry profit for the cartel will be

$$\Pi_{1+2}^* = p^* y_1^* - c_1(y_1^*) + p^* y_2^* - c_2(y_2^*) = 5.50(2.25) - 2.25 + 5.50(0) - 2(0) = \$10.125.$$

Note that these profits will be distributed according to the bargaining power each one of the firms have when negotiating. The only requirement is that each firm will be receiving at least as much as under a Cournot solution. If a firm cannot guarantee the Cournot profit, it will have no incentive to participate in a cartel agreement.