Econ 302
Suggested answers for tutorial 3

1. (a) We know that profit maximization requires

$$MC(y) = MR(y).$$

Given the cost function we get

$$MC(y) = \frac{\partial c(y)}{\partial y} \Rightarrow MC(y) = \frac{\partial}{\partial y}\left(3y^2\right) \Rightarrow MC(y) = 6y.$$

Moreover, the marginal revenue is

$$MR(y) = \frac{\partial p(y)}{\partial y}y + p(y) \Rightarrow MR(y) = \frac{\partial}{\partial y}(144-3y) \times y + (144-3y)\frac{\partial}{\partial y}y \Rightarrow$$

$$MR(y) = -3y + (144-3y) \Rightarrow MR(y) = 144 - 6y.$$

Therefore,

$$MC(y) = MR(y) \Rightarrow 6y = 144 - 6y \Rightarrow y_M = 12.$$

Given this quantity the monopoly price will be

$$p_M = 144 - 3y_M \Rightarrow p_M = 144 - 3(12) \Rightarrow p_M = \$108.$$

Finally, the profits are

$$\Pi_M = p_M \times y_M - c(y_M) \Rightarrow \Pi_M = 108(12) - 3(12)^2 \Rightarrow \Pi_M = \$864.$$

(b) Under pure competition we get

$$MC(y) = p \Rightarrow 6y = 144 - 3y \Rightarrow y_{PC} = 16.$$

Moreover, the marginal cost under the monopoly quantity is

$$MC(y_M) = 6y_M \Rightarrow MC(y_M) = 6(12) = \$72$$

Therefore, the dead weight loss is equal to

$$DWL = \left(\frac{1}{2}\right) \times (108-72) \times (16-12) = \$72.$$

(c) For the monopoly to produce the competitive quantity it must be the case that, given that the per unit subsidy reduces the marginal cost by say s dollars, the profit maximizing condition will be satisfied at $y = y_{PC}$, *i.e.*,

$$MC(y_{PC}) - s = MR(y_{PC}) \Rightarrow 6y_{PC} - s = 144 - 6y_{PC} \Rightarrow 6(16) - s = 144 - 6(16) \Rightarrow s = \$48 \text{ per unit}.$$

Given this subsidy the cost of this policy is

$$Cost\ of\ Subsidy = \$48 \times 16 = \$768.$$

The problem with this policy is more of a moral issue: in order to correct the inefficiency caused by the monopolist we subsidize her/him! Perhaps a better solution is to try to "break" the monopoly.

2. First, we need to find the total demand by horizontally summing the two individual demands. By inverting the inverse demands we get

$$p_1 = 45 - 0.5y_1 \Rightarrow y_1 = 90 - 2p_1,$$

and

$$p_2 = 30 - y_2 \Rightarrow y_2 = 30 - p_2.$$

Therefore, by adding the two demands (remember that $y = y_1 + y_2$ and, since there is no price discrimination, $p_1 = p_2 = p$) by parts we get

$$y_1 + y_2 = (90 - 2p_1) + (30 - p_2) \Rightarrow y = 120 - 3p.$$

The corresponding inverse demand (for the proper price range) is then

$$p(y) = 40 - (1/3)\,y$$

Second, we apply the profit maximizing condition for the monopolist, *i.e.*,

$$\begin{aligned} MC(y) &= MR(y) \Rightarrow \frac{\partial c(y)}{\partial y} = \frac{\partial p(y)}{\partial y} y + p(y) \Rightarrow \\ (1/3)\,y &= -(1/3)\,y + 40 - (1/3)\,y \Rightarrow y_M = 40. \end{aligned}$$

Given the optimal quantity, the monopoly price will be

$$p_M = 40 - (1/3)\,y_M \Rightarrow p_M = 40 - (1/3)\,40 \Rightarrow p_M = \$26.67.$$

Finally, we get the monopolist's profit as

$$\Pi_M = p_M \times y_M - (1/6)\,y_M^2 \Rightarrow \Pi_M = 26.67\,(40) - (1/6)\,40^2 = \$800.13$$

(b) Under price discrimination of third degree the profit maximizing conditions are now $p_1 = 45 - 0.5y_1$ and $p_2 = 30 - y_2$

$$\left\{ \begin{array}{l} MC(y_1 + y_2) = MR_1(y_1) \\ \\ MC(y_1 + y_2) = MR_2(y_2) \end{array} \right. \Rightarrow \left\{ \begin{array}{l} MC(y_1 + y_2) = \dfrac{\partial p_1(y_1)}{\partial y_1} y + p(y_1) \\ \\ MC(y_1 + y_2) = \dfrac{\partial p_2(y_2)}{\partial y_2} y_2 + p_2(y_2) \end{array} \right.$$

The first equation above yields

$$(1/3)(y_1 + y_2) = -0.5y_1 + 45 - 0.5y_1 \Rightarrow \frac{4}{3}y_1 + \frac{1}{3}y_2 = 45,$$

while the second yields

$$(1/3)(y_1 + y_2) = -y_2 + 30 - y_2 \Rightarrow \frac{1}{3}y_1 + \frac{7}{3}y_2 = 30.$$

These two equations constitute a $2x2$ system that can be solved for

$$y_1^* = \frac{95}{3},$$
$$y_2^* = \frac{25}{3}.$$

Given these optimal quantities, the corresponding prices are

$$p_1^* = 45 - 0.5y_1^* \Rightarrow p_1^* = 45 - 0.5\left(\frac{95}{3}\right) \Rightarrow p_1^* = \$29.2$$

and

$$p_2^* = 30 - y_2^* \Rightarrow p_2^* = 30 - \frac{25}{3} \Rightarrow p_2^* = \$21.67.$$

The corresponding third degree price discriminating profits are

$$\Pi_M = p_1^* \times y_1^* + p_2^* \times y_2^* - (1/6)(y_1^* + y_2^*)^2 \Rightarrow$$

$$\Pi_M = 29.2\frac{95}{3} + 21.67\frac{25}{3} - (1/6)\left(\frac{95}{3} + \frac{25}{3}\right)^2 = \$838.58$$

Note that, as expected, these profits are higher than those in case of single pricing in part (a).

(c) By dropping the low demand (i.e., $y_2 = 0$) the monopolist maximizes his profit over the first market, i.e.,

$$MC(y_1 + 0) = \frac{\partial p_1(y_1)}{\partial y_1}y + p(y_1) \Rightarrow (1/3)y_1 = -0.5y_1 + 45 - 0.5y_1 \Rightarrow y_1^* = 33.75$$

The corresponding price is then

$$p_1^* = 45 - 0.5y_1^* \Rightarrow p_1^* = 45 - 0.5(33.75) \Rightarrow p_1^* = \$28.13$$

and the resulting profits are

$$\Pi_M = p_1^* \times y_1^* - (1/6)(y_1^* + y_2^*)^2 \Rightarrow \Pi_M = 33.75(28.13) - (1/6)(33.75)^2 = \$759.38$$

To conclude, in this example the best solution (out of the three examined) is for the monopolist to price discriminate in third degree. If he does not have that option (because arbitrage is possible, for example) he should serve the entire market.