| Objective function:**2 marks** |
|---|

# PART 2 (30 marks + 2 Bonus Marks)

**Questions 1 to 4 refer to the problem below**

Karma Computers produces two computer models: Standard and Deluxe. A standard model is produced by assembling a single Disk Drive and a Standard Chassis. A Deluxe model is produced by assembling two disk drives and a Deluxe Chassis.

| Computer Model | Disk Drives | Standard Chassis | Deluxe Chassis |
|---|---|---|---|
| Standard | 1 | 1 | |
| Deluxe | 2 | | 1 |

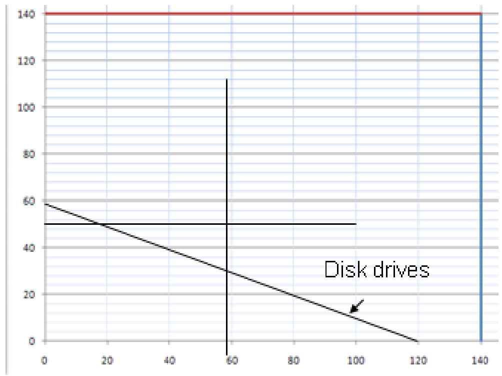
Disc drive costs $2 each, Standard Chassis costs $3 each and Deluxe Chassis costs $4 each. Labour cost for each Standard Model is $10 while that of the Deluxe Model is $15 each. Assume that there are no other costs to produce the two computer models.

The Standard model has a net profit of $3 while the Deluxe model has a net profit per unit of $4. The current inventory consists of 60 Standard Chassis, 50 Deluxe Chassis, and 120 Disk Drives

Let
X1= Quantity of Standard Model
X2 = Quantity of Deluxe Model
The problem formulation is summarized below: (you may use the grid below to facilitate your work but it would not be graded)

Max: Z =3X1 + 4X2
St:
Disc Drives: X1+2X2 <= 120
Standard Chassis: X1 <= 60
Deluxe Chassis X2 <= 50

1. The optimum solution to this problem is
   a) X1 = 60, X2 = 50
   **b) X1 = 60, X2 = 30**
   c) X1= 30, X2 = 50
   d) X1 = 20, X2 = 50




2. If Karma has the opportunity of buying 10 more units each of Standard Chassis and Deluxe Chassis (assume that there no more supply of Disk Drives), which should they buy in order to maximize profit?
   a) neither
   b) both Standard and Deluxe Chassis
   **c) Standard Chassis To produce one more Std we give up a deluxe, get two Disk drives, make two standard. So although the profit from one standard is less than from one deluxe, we get two standards by giving up one deluxe, so we give up 4 and gain 6.**
   d) Deluxe Chassis

3. If Karma has the opportunity of buying any number of Disk Drives (assume that there are no more supplies of Standard and Deluxe Chassis beyond what is currently in inventory) at the regular price how many should they buy to maximize profit?
   a) 20
   **b) 40 They have 20 unused Deluxe Chassis left; fill 'em up with 2 disks each…**
   c) 80
   d) 100

4. By intensive marketing and aggressive pricing the objective function (the profit for each model) has now changed such that buying one more of Deluxe Chassis is better than buying one more Standard Chassis. This must mean that the new optimum solution must consist of

   a) X1 = 0 and X2 = 50
   b) X1 =60
   c) X2 =0
   **d) X2 =50**

**Questions 5 to 8 refer to the problem below.**

| | Wheat | Rice | Corn | Requirements per 12-Ounce Box |
|---|---|---|---|---|
| Protein (grams per ounce) | 4 | 2 | 2 | At least 27 grams |
| Carbohydrates (grams per ounce) | 20 | 25 | 21 | At least 240 grams |
| Calories per ounce | 90 | 110 | 100 | No more than 1.260 calories |
| Cost per ounce | $.03 | .05 | .02 | |

Formulate an LP model for this problem that will determine the optimal quantities of wheat, rice, and corn per box that will achieve the requirements at minimum most.

minimize $Z = .03x_1 + .05x_2 + .02x_3$

subject to

| | |
|---|---|
| Protein | $4x_1 + 2x_2 + 2x_3 \geq 27$ grams |
| Carbohydrates | $20x_1 + 25x_2 + 21x_3 \geq 240$ grams |
| Calories | $90x_1 + 110x_2 + 100x_3 \leq 1{,}260$ calories |
| Box size | $x_1 + x_2 + x_3 = 12$ ounces |
| | $x_1, x_2$, and $x_3 \geq 0$ |

5. The dietician changes the required protein from 27 grams per 12 ounce bar to 29 grams per 12 ounce bar. Assume that the protein constraint is a binding constraint.

   a) the feasible region will increase, and the objective function value may decrease
   b) the feasible region will decrease, and the objective function value may decrease
   c) the feasible region will increase, and the objective function value may increase
   **d) the feasible region will decrease, and the objective function value may increase**

6. The dietician adds a constraint that requires the sugar content to be at least 12%. The sugar content of wheat is 11%, of rice 9% and of corn 14%. The dietician recognizes that the constraint is:

$$\frac{11x_1 + 9x_2 + 14x_3}{x_1 + x_2 + x_3} \geq 12$$

   The correct form, with variables on the left-hand side and constant term on the right-hand side is:

   a) $11x_1 + 9x_2 + 14x_3 \geq 12$
   b) $-x_1 - 3x_2 + 2x_3 \leq 0$
   **c) $x_1 + 3x_2 - 2x_3 \leq 0$**
   d) $x_1 + 3x_2 - 2x_3 \geq 0$

7. Adding a constraint of the form: $2x_1 + x_2 + x_3 \geq -5$ to the dietician's problem above could:

a) make the problem infeasible.
b) produce multiple optimal solutions.
**c) be redundant.**
d) all of the above.

8. Adding a constraint of the form: $x_1 + x_2 + x_3 \leq 0$ to the dietician's problem above could:

**a) make the problem infeasible.**
b) produce multiple optimal solutions.
c) be redundant.
d) all of the above.

9. For a constraint, the Allowable Increase column in the sensitivity report shows

a) the allowable increase in the objective value
**b) the allowable increase in the RHS value of the constraint for which the current optimal corner point remains optimal**
c) the allowable increase in the shadow price
d) the amount the objective function value will change if the RHS value is increased one unit

10. After solving an LP problem, you discover that there are two optimum corner point optimum solutions:

a) this implies that there are two possible objective function values
**b) this implies that there are infinite number of optimum solutions**
c) the problem is a maximization problem
d) the problem is a minimization problem

11. If we increase a constraint RHS value beyond the allowable increase value shown in the sensitivity report, this means

a) there will not longer be a feasible solution for the problem
b) **the problem needs to be solved again to get a new sensitivity report** such a change should never be made
c) the shadow price in the report is still valid

12. Surplus refers to the

a) LHS value of a ≥ constraint.
b) **difference between the LHS and RHS values of a ≥ constraint**
c) difference between the LHS and RHS values of a ≤ constraint
d) RHS value of a <= constraint

13. In LP a constraint can be relaxed until

a) it becomes binding
**b) it becomes redundant**
c) it becomes difficult to solve a problem
d) it becomes the new solution

14. Goal programming

a) requires only that you know whether the goal is direct profit maximization or cost minimization
b) **allows you to have multiple goals, with or without priorities**
c) is an algorithm with the goal of a quicker solution to a pure integer programming problem
d) is an algorithm with the goal of a quicker solution to a IP problem

15. In a goal programming problem

a) deviational variables can be positive or negative
b) **only deviational variables appear in the objective function**
c) all goals are equally important
d) all goals are important but can be ignored

### Questions 16 to 19 refer to the problem below

Beady Emm was looking around his sparse pantry trying to figure out what he could eat that would be nutritionally balanced but also low cost. He used a "diet problem" website[1] and selected the foods he found in his kitchen (partial list shown):

| Click the foods you are willing to eat | | | | | | | |
|---|---|---|---|---|---|---|---|
| | **Foods** | **Price/Serving** | **Serving Size** | | **Foods** | **Price/Serving** | **Serving Siz** |
| ☐ | Frozen Broccoli | $0.16 | 10 Oz Pkg | ☐ | Carrots,Raw | $0.07 | 1/2 Cup Shred |
| ☑ | Celery, Raw | $0.04 | 1 Stalk | ☐ | Frozen Corn | $0.18 | 1/2 Cup |
| ☐ | Lettuce,Iceberg,Raw | $0.02 | 1 Leaf | ☑ | Peppers, Sweet, Raw | $0.53 | 1 Pepper |
| ☑ | Potatoes, Baked | $0.06 | 1/2 Cup | ☑ | Tofu | $0.31 | 1/4 block |

"0.01 of a chocolate chip cookie and 2.25 servings of Cheerios?!" Beady exclaimed, "I'll never be able to survive on that!" He downloaded the information from the website and build his own spreadsheet to verify these findings. "Yep, the webpage was right. My total cost is **$1.69**."

[1] Really! Its at: http://www-neos.mcs.anl.gov/CaseStudies/dietpy/WebForms/table.html

Adjustable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $B$3 | X1-Raw Celery Stalk | ? FV_Celery ? | 0.01 | 0.04 | 1E+30 | 0.01 |
| $C$3 | X2-Red Pepper | 0.00 | ? RC_Pepper ? | 0.53 | 1E+30 | 0.39 |
| $D$3 | X3-Baked Potatoes | 4.01 | 0.00 | ? OC_Potato? | 0.08 | 0.03 |
| $E$3 | X4-Tofu | 0.00 | 0.34 | 0.31 | ?AI_Tofu ? | 0.34 |
| $F$3 | X5-Banana | 0.00 | 0.04 | 0.15 | 1E+30 | ? AD_Banana ? |
| $G$3 | X6-Bagel | 0.00 | 0.14 | 0.16 | 1E+30 | 0.14 |
| $H$3 | X7-Apple Pie | 0.00 | 0.13 | 0.16 | 1E+30 | 0.13 |
| $I$3 | X8-Chocolate Chip Cooki | 0.01 | 0.00 | 0.03 | 0.02 | 0.01 |
| $J$3 | X9-Skim Milk | 4.37 | 0.00 | 0.13 | 0.03 | 0.01 |
| $K$3 | X10-Cap'n Crunch Cerea | 0.00 | 0.38 | 0.31 | 1E+30 | 0.38 |
| $L$3 | X11-Cheerios cereal | 2.25 | 0.00 | 0.28 | 0.04 | 0.16 |
| $M$3 | X12-Special K cereal | 0.00 | 0.13 | 0.38 | 1E+30 | 0.13 |
| $N$3 | X13-Peanut Butter | ?FV_Peanut_Butter? | 0.00 | 0.07 | 0.04 | 0.07 |

17. The value of OC_Potato is ________________

a) 0.00
b) 0.01
c) 0.04
d) 0.05
**e) 0.06 From the table above…**

18. The value of AI_Tofu is ______________________

a) 0.00
b) 0.01
c) 0.34
d) 0.031
**e) 1E+30**

## Question 19 below is for 1 Bonus Mark (1 extra mark)

19. Assume that the Final Value FV-Celery is 0.01, then the value of FV_Peanut_butter is

a) 1.59
b) 2.59
c) **3.59**
d) 4.59
e) Can not be determined

### Questions 20 to 24 refer to the following

The diet problem has constraints for the nutrients (calcium, calories, carbohydrates, etc) that are written as:

$$MIN\_VALUE \leq A_1X_1 + A_2X_2 + A_3X_3 + \ldots + A_{12}X_{12} + A_{13}X_{13} \leq MAX\_VALUE$$

In the case of calcium for example, the (partial) constraint looks like:

$$800 \leq 16X_1 + 6.7X_2 + 22.7X_3 + \ldots + 8.2X_{12} + 13.1X_{13} \leq 1600$$

The $A_1$, $A_2$, etc. are placeholders for the constraint coefficients and, like the minimum and maximum values, vary by each nutrient.

Use ONLY the following values to fill in the missing shadow prices for the dietary fiber and iron constraints. You can use a value more than once. Some values might not be used at all.

Constraints (partial report shown)

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $P$22 | Dietary_Fiber-max | 25 | ? SP_Fibre_Max ? | 100 | 1E+30 | 75 |
| $P$11 | Dietary_Fiber-min | 25 | ? SP_Fibre_Min ? | 25 | 0 | 5 |
| $P$27 | Iron-max | 30 | ? SP_Iron_Max ? | 30 | 4 | 0 |
| $P$16 | Iron-min | 30 | ?SP_Iron_Min ? | 10 | 20 | 1E+30 |

The only possible answers to questions 19 to 22 below are shown in the Table below. Write the letter that corresponds with the correct value in each case. (**Enter the letter in your Scranton for the corresponding question**)

| Possible Answers | A | B | C | D | E |
|---|---|---|---|---|---|
| Value | -Infinity | -0.0158 | 0.000 | 0.0255 | + Infinity |

20. The Value of SP_Fibre_Max is ______**C**______

21. The Value of SP_Fibre_Min is _______**D**_Constraint is binding, ≤, so increase will harm the Min Cost objective function, so shadow price must be positive

22. The Value of SP_Iron_Max is ______**B**__Constraint is binding, ≥, so increase RHS will harm the Min cost objective function, so shadow price must be negative

23. The Value of SP_Iron_Min is _____**C**______

24. Beady really craves Apple Pie. What is the impact on his overall diet's cost if he insists on in.cluding one serving of pie?

**a) Increase by 0.13**
b) Decrease by 0.13
c) Increase by 0.16
d) Decrease by 0.16
e) Cannot determine

**Question 25and 26 refer to the problem and output below**

Westco Furnishing manufactures 4 types of furniture: Sofa, Chair, Tables and Bookshelves. The profit margin from each type of furniture are shown as the objective function coefficients. Making each type of furniture requires wood and labour (Cutting, Assembly ) polish and drying. An LP to determine the quantity of each type of furniture that would maximize profit was solved and the result shown in the Table below.

Target Cell (Max)

| Cell | Name | Original Value | Final Value |
|---|---|---|---|
| $M$13 | Objective Function Value | 0 | 96 |

Adjustable Cells

| Cell | Name | Final Value | Reduced Cost | Objective Coefficient | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $H$11 | Sofa | 12 | 0 | 4 | 8 | 16 |
| $I$11 | Chair | 0 | -7 | 5 | 7 | 1E+30 |
| $J$11 | Table | 0 | -4 | 6 | 4 | 1E+30 |
| $K$11 | Bookshelf | 4 | 0 | 12 | 1E+30 | 5.33 |

25. If the profit margin of chair increases to $10 and that of Sofa increases to $10. What would happen to the current solution.

a) The new Objective Function Value would not change
b) The New objective Function Value would be lower
**c) The Solution may change and need to be re-solved to get a new solution**
**100% rule is violated…**
**d) The solution would not change and the Objective Function Value would remain the same (if student knows that 100% rule does not apply when shadow price/reduced cost = zero)**

26. If the profit margin of chair increases to $10 and that of Table increases to $9. What would happen to the current solution.

a) The Objective Function Value would change
b) The New Objective Function Value would be lower
**c) The solution may change and the problem must be re-solved for the new solution 100% rule violated…**
**d) The solution would not change and the Objective Function Value would remain the same (if student knows that 100% rule does not apply when shadow price/reduced cost = zero)**

**Questions 27 and 28 refer to this output**

Constraints

| Cell | Name | Final Value | Shadow Price | Constraint R.H. Side | Allowable Increase | Allowable Decrease |
|---|---|---|---|---|---|---|
| $M$15 | Wood(SqaureMeter) | 16 | 16 | 16 | 0.19 | 2.67 |
| $M$16 | Cutting (Hours) | 92 | 0 | 120 | 1E+30 | 28 |
| $M$17 | Assembly(Hours) | 96 | 0 | 100 | 1E+30 | 4 |
| $M$18 | Polish(Grams) | 68 | 0 | 40 | 28 | 1E+30 |
| $M$19 | Drying(hours) | 80 | -2 | 80 | 16 | 1.33 |

27. If the amount of wood decreases to 15 and the number of drying hours increases by 5, what would happen to the current solution? (OFV=Objective Function Value)

a) The solution would remain the same and the OFV would increase by 26
**b)The solution would remain the same and the OFV would decrease by 26** MAX Problem; OFV new = OFV old + [(-1)(16) + (+5)(-2)] = OFV old – 26…
c) The solution would remain the same and the OFV would decrease by 6
d) The solution would change and the problem must be resolved

28. If the cutting hours is decreased to 100 and the polish amount increases to 67 grams and the assembly hours increased by 20, what would happen to the current solution and the objective function

a) The Objective Function Value would not change but the solution would change
**b) The Objective Function Value and the solution would change**
c) The Objective Function Value and solution would not change
**This solution is also correct.**
d) The solution would not change but the Objective Function Value would change

## Question 29 refers to the following

If Xi is a binary variable that is equal to 1 if product i is produced and zero otherwise, interpret the constraints in problem 29 below:.

29. The constraint $3X1 \leq X2 + X3 + x4$ means that
**a) Product 1 can be produced only if products 2, 3 and 4 are produced**
b) If products 2,3,and 4 are produced then product 1 must be produced
c) Products 2,3, and 4 must not be produced in order that product 1 can be produced
d) Product 1 is not as important as products 2, 3, and 4

**Question 30 to 31 refer to the problem below**

An AP Finance Investment Co is considering investment Columbia, Peru, Brazil, Argentina, and Venezuela. AP is interested in minimizing investment risks and wants to choose where to invest carefully. The Table below shows the Decision Variables to be used for the problem.

| 0,1 variables | Interpretation | Amou nt | Interpretation |
|---|---|---|---|
| YC | =1 if investment in Columbia, zero otherwise | XC | Amount invested in Columbia |
| YP | =1 if investment in Peru, zero otherwise | XP | Amount invested in Peru |
| YB | =1 if investment in Brazil, zero otherwise | XB | Amount invested in Brazil |
| YA | =1 if investment in Argentina, zero otherwise | XA | Amount invested in Argentina |
| YV | =1 if investment in Venezuela, zero otherwise | XV | Amount invested in Venezuela |

30. If they invest in Brazil they should not invest in Argentina or Venezuela
(M is a very large number.)

a) YB ≥ YA and YB ≥YV
**b) YB+YA ≤ 1 and YB+YV ≤ 1**
c) YA+YV ≤ M(YB)
d) YA+YV+YB ≤ 1 No, this will not allow investing in Argentina and Venezuela.

**Q31 is a BONUS QUESTION!**

31. If they invest in Brazil, the amount should be at least $500,000.00
(M is a very large number.)

a) XB ≥ 500,000.00 and XB ≤ M(YB)
**b) XB ≥ 500,000.00YB and XB ≤ M(YB)**
c) XB ≥ 500,000.00 and XB≥ M(YB)
d) XB +YB ≥ 500,000.00

**Question 32 refers to this problem.**

The Lightwell Chandelier Company of British Columbia produces two expensive chandeliers. The products are popular with customers who are renovating old homes. Both products require a two-step process, first for wiring, second for assembly.

The Golden Brandy Chandelier requires two hours to wire and six hours to assemble. The Cobblestone Marcella model requires three hours to wire and five hours to assemble.

Lightwell has four goals that it wants to accomplish in choosing how many Golden Brandy (GB) and how many Cobblestone Marcellas (CM) to produce. These goals are equally important to the company.

1) The company wants to achieve a profit of $3000.
2) It wants to avoid underutilization of the wiring department.
3) It wants to avoid overtime in the assembly department.
4) They also want to meet a contract requirement to produce seven Cobblestone Marcellas.

Define the deviational variables as:

$u_1$= underachievement of profit target
$v_1$=overachievement of profit target
$u_2$ =idle time in the wiring department (underutilization)
$v_2$ =over time in the wiring department (overutilization)
$u_3$=idle time in the assembly department (underutilization)
$v_3$ =over time in the assembly department (overutilization)
$u_4$ =underachievement of Cobblestone Marcella goal
$v_4$ =overachievement of Cobblestone Marcella goal

32. The correct objective function for the problem is:

a) **minimize** $u_1 + u_2 + v_3 + u_4 + v_4$
b) minimize $u_1 + v_1 + u_2 + v_2 + u_3 + v_3 + u_4 + v_4$
c) minimize $u_1 + u_2 + v_3 + u_4$
d) maximize $u_1 + v_1 + u_2 + v_2 + u_3 + v_3 + u_4 + v_4$
e) none of the above