CVG2132 – FUNDAMENTALS OF ENVIRONMENTAL ENGINEERING

Homework 3: Fall 2015

Due Date: Nov. 10, 2015 (4:00pm) – Drop box « CVG 2132 », Mezzanine A (0.5) CBY

Question 1: The concentration of species C was measured as a function of time during a chemical reaction. The observed concentration of species C at various time intervals is presented below.

Answer the following questions: **(a)** Determine the reaction order of substance C and rate constant k_C (show all work and the evaluations of every kinetic order) **(b)** Is species C being removed or produced? **(c)** If the half-life of a chemical compound A is 30 days under these same conditions, determine the first-order removal rate constant of species A, k_A.

Time (minutes)	Concentration C (mg/L)
0	200
1	142
2	111
3	90
4	77
5	67

SOLUTION :

(a) Determine the reaction order of substance C and rate constant k_C (show all work and the evaluations of every kinetic order)

i) Check if it is a zero order reaction $r = \frac{dC}{dt} = -k$

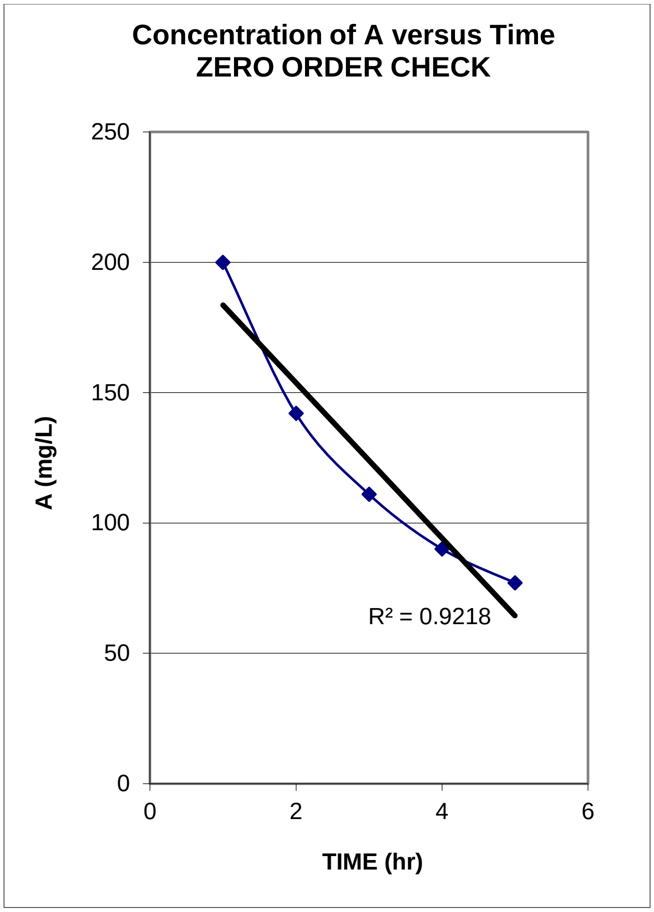
Rearrange this expression to $dC = -k \cdot dt$

And integrate from time = 0 and concentration Co to time t and concentration C one gets

$$C\Big|_{C_0}^{C} = -k \cdot t\Big|_{0}^{t}$$

This expression solves to $C = C_0 - k \cdot t$

So plot C versus time to check if the reaction is zero order



Since the data deviates from a straight line, it is not described by a zero order reaction. Another order will likely describe the data better.

Now try a first order reaction $r = \frac{dC}{dt} = -k \cdot C$

Rearrange this expression to $\frac{dC}{C} = -k \cdot dt$

And integrate from time = 0 and concentration Co to time t and concentration C one gets

$$\int_{C_0}^{C} \frac{dC}{C} = -k \cdot \int_{t_0}^{t} dt$$

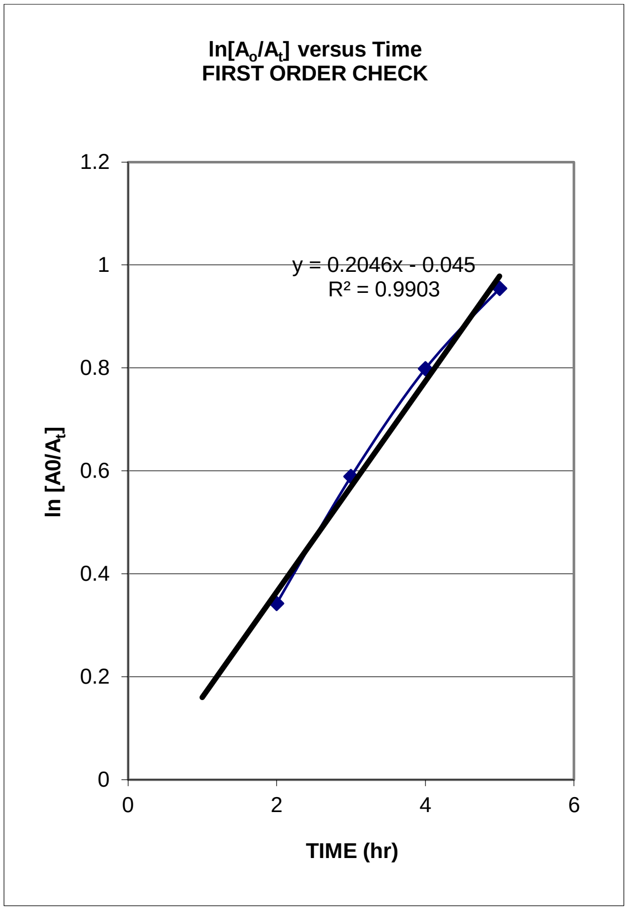
$$\ln C \Big|_{C_0}^{C} = -k \cdot t \Big|_{0}^{t}$$

The linear type solutions for this rate expression are

a) $\ln\left(\frac{C}{C_0}\right) = -k \cdot t$ and

b) $\ln\mathrm{C} - \ln C_0 = -k \cdot t$

So calculate ln C/C_0 and plot versus time to see if the data yields a straight line.



So the first order kinetics describes this data better but there mmay be a better fitting model. Let's try a second order reaction.

$$r = \frac{dC}{dt} = -k \cdot C^2$$

Rearrange this expression to $\frac{dC}{C} = -k \cdot dt$

And integrate from time = 0 and concentration Co to time t and concentration C one gets

$$\int_{C_0}^{C} \frac{dC}{C^2} = \int_{C_0}^{C} C^{-2} dC = -k \cdot \int_{t_0}^{t} dt$$

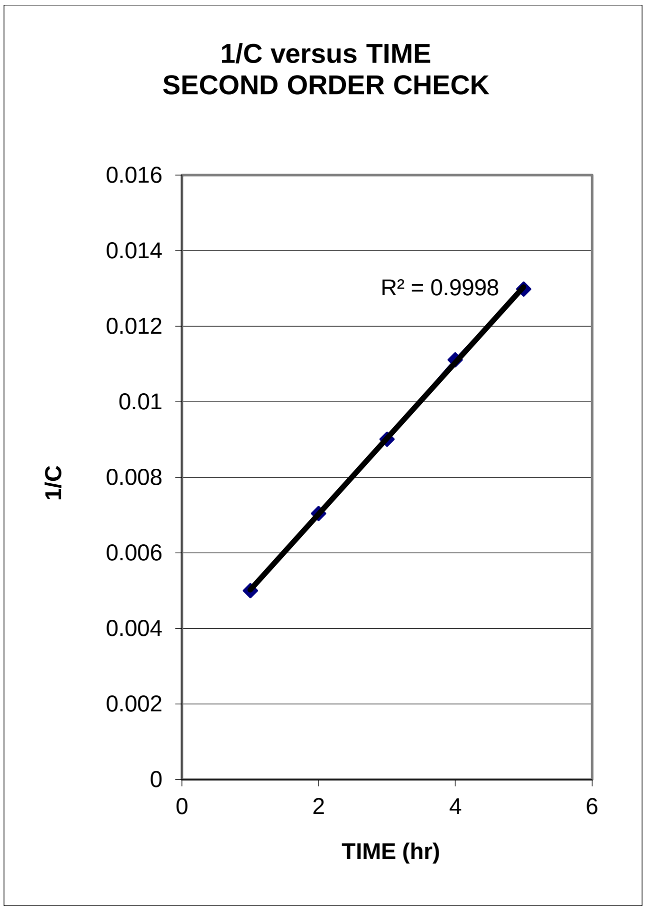
$$\frac{1}{-1} C^{-1} \Big|_{C_0}^{C} = -k \cdot t \Big|_{0}^{t}$$

The linear type solutions for this rate expression is

$$\frac{1}{C} = \frac{1}{C_0} + k \cdot t$$

So plot the data in a 1/C versus t graph

TIME h (1)	Conc. (mg/L) (2)	1/(A)
0	200	0.005
1	142	0.007042
2	111	0.009009
3	90	0.011111
4	77	0.012987
5	67	0.014925



This yields an excellent fit. So this reaction must be following a second order reaction.

K= slope of line = $\frac{0.014925\,\frac{L}{mg} - 0.005\,\frac{L}{mg}}{5-0\ h} = 0.001985\,\frac{L}{mg \cdot h}$

Answer to part a) This is a second order reaction with a rate constant of 0.001985 min^{-1}

(b) Is species C being removed or produced?

Answer to part b) C is being **removed**.

(c) If the half-life of a chemical compound A is 30 days under these same conditions, determine the first-order reaction rate constant of species A, k_A.

$\ln 0.5 = -k_A t_{1/2}$

$\mathbf{k_A} = 0.693/t_{1/2}$ = **0.023 1/d**

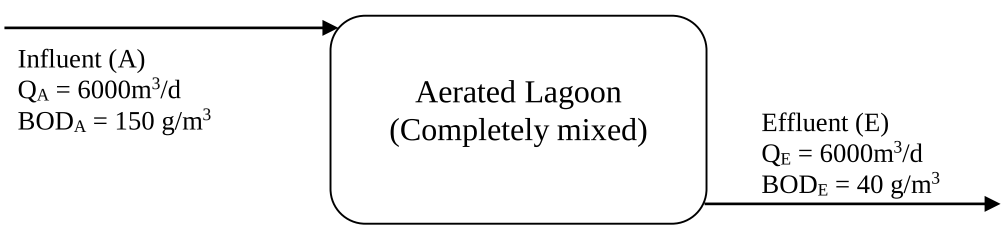
Answer to part C) $\mathbf{k_A}$ = **0.023 1/d**

Question 2: (from 2013 midterm) The town of Silver Lake treats its 6000 m^3/d municipal sewage flow using a completely mixed aerated lagoon. BOD is one of principal parameters used to measure the strength of biodegradable organic compounds in sewage. The raw wastewater has a BOD concentration of 150 g/m^3 and the biological reactions in the aerated lagoon decrease the BOD concentration to 40 g/m^3 by the time it exits the lagoon. The biodegradation occurs via a second order reaction with rate constant of 0.02 $m^3/(g{\cdot}d)$.

TASKS:

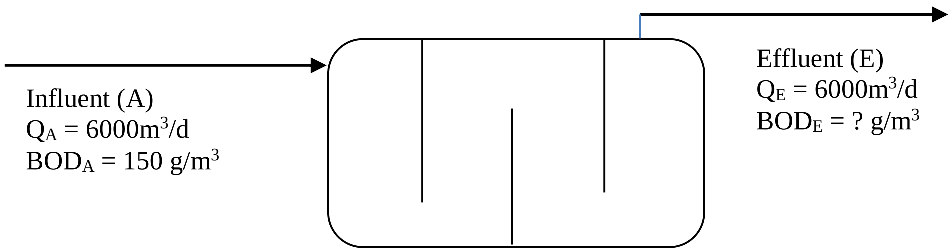
<u>Showing all your derivations and work to</u>

a) Determine the effective volume of the lagoon.



b) The plant needs to be upgraded to meet new federal BOD effluent regulation, they require that effluent BOD concentration should be less than 25 g/m^3. The proposed plan is to add plastic walls within the lagoon so as to make the flow pattern approach that of a PFR. Assume the ideal PFR reactor performance can be achieved, and the volume of the walls is negligible. **<u>Determine the effluent</u>**

BOD concentration of the modified lagoon. If you cannot determine the answer of part a) assume the effective volume of the aerated lagoon is 50000 m^3.



SOLUTION

a) Determine the volume of the CSTR?

Q_A =6000 m^3/d
BOD_A =150 g/m^3
BOD_E =40 g/m^3
Second order reaction, k =0.02 m^3/(g·d)
So

$$\frac{dBOD}{d\theta} = r_{BOD} = -k \cdot BOD^2$$

As there is no information indicating changes with time assume steady state conditions prevail. (1 pt)

As there is no information regarding the densities of the water in the various streams, assume they all have a density of 1000 kg/m^3.

CSTR Analysis for the BOD in the aerated lagoon

$$\text{Accumulation} = \frac{d(V \times BOD)}{dt} = (Q_A \times BOD_A) - (Q_E \times BOD_E) - (V_{CSTR} \times r)$$

Accumulation = 0 because of the steady-state conditions
so

$$0 = (Q_A \times BOD_A) - (Q_A \times BOD_A) - (V_{CSTR} \times k \times BOD_E^2)$$

and

$$0 = \left(6000\frac{m^3}{d} \times 150\frac{g\ BOD}{m^3}\right) - \left(6000\frac{m^3}{d} \times 40\frac{g\ BOD}{m^3}\right) - \left(V_{CSTR} \times 0.02\frac{m^3}{g \times d} \times \left(40\frac{g\ BOD}{m^3}\right)^2\right)$$

And V= 41250 m^3

Answer to part a: V= 41250 m^3

b) **Determine the effluent BOD concentration of the modified (PFR) lagoon.**

Θ_{PFR} = V_{PFR} / Q = (41250 m^3) / (6000 m^3/d) = 6.875 d

Derive the expression for a steady-state second order PFR

$$\frac{dC_A}{d\theta} = r_A = -k \cdot C_A^2$$

Collect the θ terms in the R.H.S and the C_A terms in the L.H.S.

$$\frac{dC_A}{{C_A}^2} = -k \cdot d\theta$$

Integrate from θ = 0 to θ and C_A from C_{Ain} to C_{Aout}

$$\int_{C_{Ain}}^{C_{Aout}} C_A^{-2}\, dC_A = -k \cdot \int_0^{\theta} d\theta$$

$$\frac{1}{-1} C_A^{-1} \Big|_{C_{Ain}}^{C_{Aout}} = -k \cdot \theta \Big|_0^{\theta}$$

so

$$BOD_E^{-1} - BOD_A^{-1} = -k \cdot (-1) \cdot \theta$$

and

$$BOD_E = \frac{1}{\left[\left(\frac{1}{BOD_A}\right) + (k \cdot \theta)\right]} = \frac{1}{\left[\left(\frac{1}{150\,\frac{g}{m^3}}\right) + \left(0.02\,\frac{m^3}{g \cdot d} \cdot 6.875d\right)\right]} = 13.25\,\frac{g}{m^3}$$

Answer to part b) BOD_E = 13.25 g/m^3

Note if V= 50000 m^3/d, Θ_{PFR} = 20 d, C_E = 9.57 g/m^3

Question 3: (from midterm 2012) The Town of Havasplitflo waste water treatment plant has 2 flows entering the plant. A main flow Q of $4m^3/s$ and a minor flow Qm of $0.5m^3/s$.

The main flow, Q splits at the plant headworks (point A) (See Figure Below) with 30 percent of Q going in one direction the other 70 percent the other direction. Tracer studies have shown that the flow from A-E (0.3Q) behaves as a single plug flow reactor (PFR) with a volume of $30,000m^3$.

The other flow (0.7Q) from A-B-C-D-E can be characterized by a PFR (15,000 m^3) followed by continuous flow stirred tank reactor (CFSTR) ($135,000m^3$) followed by a PFR ($10,000m^3$). The second minor flow, Qm enters the treatment plant by flowing into the section behaving as a CFSTR (See Figure).

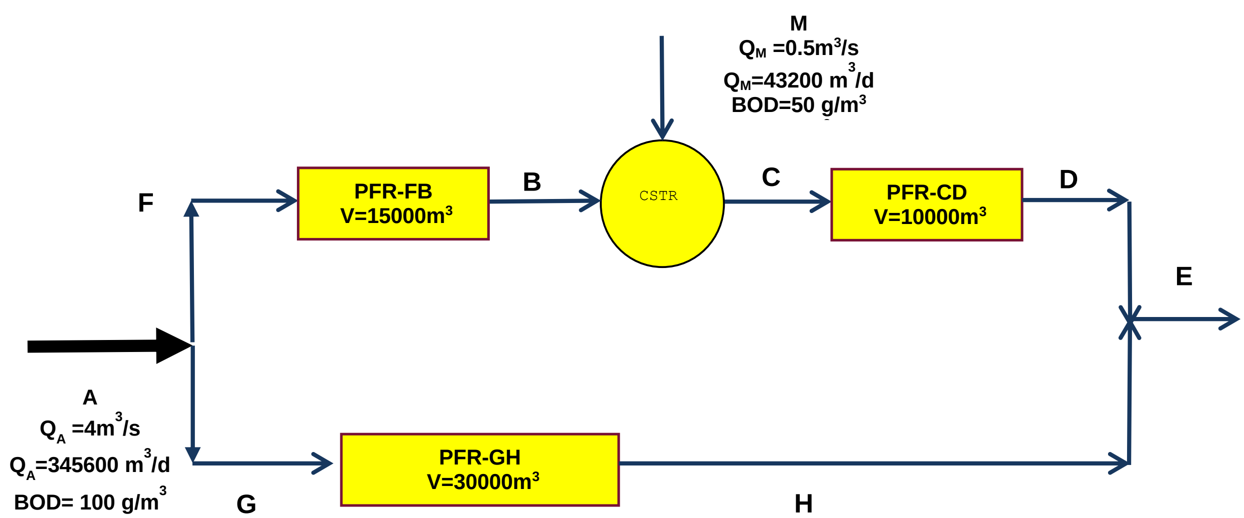
For steady-state conditions, determine the concentration of reactant (BOD) at point E for the following conditions:

$r_{BOD} = -kC_{BOD}$ (1st order reaction)
$k = 0.5\ d^{-1}$
$C_{BOD} = 50\ g/m^3$ (minor flow Qm)
$C_{BOD} = 100\ g/m^3$ (main flow Q)

SKELETON SOLUTION

As there is no information indicating changes with time assume steady state conditions prevail.

As there is no information regarding the densities of the water in the various streams, assume they all have a density of 1000 kg/m^3.



$Q_F = 0.7 * Q_A = 241920\ m^3/d$
$Q_G = 0.3 * Q_A = 103680\ m^3/d$

Because of the A-F-G split, $BOD_G = BOD_F = BOD_A = 100\ g/m^3$

Total Mass Balance around PFR-FB

$$\text{Accumulation} = \Sigma \text{Mass In} - \Sigma \text{Mass Out}$$

$$0 = M_F - M_B$$

$$0 = (\rho_F \cdot Q_B) - (\rho_B \cdot Q_B)$$

$$\text{since } \rho_F = \rho_B = 1000 \frac{kg}{m^3}$$

$$0 = (Q_F) - (Q_B)$$

$$Q_B = Q_F = 241920 \frac{m^3}{d}$$

Total mass Balance around the CSTR
Similarly it yields $Q_C = 285120\ m^3/d$. You are expected to write the complete mass balance in each case

Total Mass Balance around PFR-CD
Similarly it yields $Q_D = 285120\ m^3/d$

Total Mass Balance around PFR-GH
Similarly it yields $Q_H = 103680\ m^3/d$

Total Mass Balance for the system or around the mixing point
It yields $Q_E = 388800\ m^3/d$

Calculate the hydraulic retention times $\theta = \frac{V}{Q}$

$\Theta_{PFR\text{-}FB} = V_{PFR\text{-}FB} / Q = (15000\ m^3) / (241920\ m^3/d) = 0.062\ d$

Similarly
$\Theta_{PFR\text{-}GH} = 0.2893\ d$

$\Theta_{PFR\text{-}CD} = 0.0351\ d$

Derive the expression for a steady-state first order PFR

$$\frac{dC_A}{d\theta} = r_A = -k \cdot C_A$$

Collect the θ terms in the R.H.S and the C_A terms in the L.H.S.

$$\frac{dC_A}{C_A}$$

Integrate from θ = 0 to θ and C_A from C_{Ain} to C_{Aout}

$$\int_{C_{Ain}}^{C_{Aout}} \frac{dC_A}{C_A} = -k \cdot \int_0^{\theta} d\theta$$

$$lnC_A \Big|_{C_{Ain}}^{C_{Aout}} = -k \cdot \theta \Big|_0^{\theta}$$

so

$$ln\frac{C_{Aout}}{C_{Ain}} = -k \cdot \theta$$

and

$$\frac{C_{Aout}}{C_{Ain}} = e^{-k \cdot \theta}$$

Apply steady-state first order analysis for the BOD in PFR-GH

$C_H = C_G * \exp(-k * \Theta_{PFR\text{-}GH}) = (100\ g/m^3) * \exp(-0.5\ d^{-1} * 0.2893\ d) = 86.53\ g/m^3$

Apply steady-state first order analysis for the BOD in PFR-FB

Similarly obtain

$C_B = C_F * \exp(-k * \Theta_{PFR\text{-}FB}) = 96.95\ g/m^3$

Apply the BOD steady state CSTR analysis for the CSTR

$$Accumulation = \frac{d(V \cdot BOD)}{dt} = (Q_B \cdot BOD_B) + (Q_M \cdot BOD_M) - (Q_C \cdot BOD_C) - (V_{CSTR} \cdot r)$$

$$Accumulation = 0 \quad \text{because of the steady state conditons}$$

$$0 = \left(241920\frac{m^3}{d} \cdot 96.95\frac{g}{m^3}\right) + \left(43200\frac{m^3}{d} \cdot 50\frac{g}{m^3}\right) - \left(285120\frac{m^3}{d} \cdot BOD_C\right) - \left(V_{CSTR} \cdot 0.5 \cdot BOD_C^{\ 1}\right)$$

Solve for C_C = 72.64 g/m^3

Apply the steady-state first order analysis for the BOD in PFR-CD

$C_D = C_C * \exp(-k * \Theta_{PFR-CD}) = 71.38\ g/m^3$

Obtain CD

<u>Conduct BOD mass balance for the mixing point D-H-E</u>

$$Accumulation = \Sigma Mass\ BOD\ In - \Sigma Mass\ BOD\ Out$$

$$0 = (Q_D \cdot BOD_D) + (Q_F \cdot BOD_F) - (Q_E \cdot BOD_E)$$

Solve for $C_E = BOD_E = 75.4\ g/m^3$

ANSWER
$BOD_E = 75.4\ g/m^3$