

Lecture 13: Review Sample Midterm

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About the Midterm

- Closed book (cover Lectures 1-10), 80-min (11:30 am - 12:50 pm) in class on Wednesday, **Nov 02**.
- 12 multiple-choice questions (12 marks in total); each question only has one correct answer.
- Sign and hand in the 1st page only.
- More than one versions...
- Formula sheet, Z-table, and scrap paper will be provided. **In addition, you can bring one self-prepared formula sheet (A4 size, double-sided).**
- Bring your Faculty-standard calculator(s): Texas Instruments TI-30 and TI-34, Casio FX-260 and Casio FX-300.
- Special office hours on Tuesday (**Nov 01**): 10:00-12:00, 1:30-5:00 in KED B07-C. No office hour on Wednesday (Nov 02).

About the Midterm

- Carefully review our lecture notes, the examples discussed in class, Assignments 1-2, and the sample midterm.
- Extra exercises are available at:
<http://aix1.uottawa.ca/~glamothe/questions/introbioEN.html>
- On average, you should spend 7 min on each question. It is not a bad idea to “skip” some hard questions.
- If a question asks you to find an incorrect statement, read all statements first and choose/check the one that is most suspicious.
- You won't be asked to solve/compute integrals.

2014 Midterm: Question 1

Solution: Let Y be the allele for regular yields and y for high yields. Let H be the allele for normal height and h for dwarfism.

- The dwarf plant has genotype $Yyhh$ and the other plant has genotype $YyHh$.
- Their offspring has possible genotypes:

$YYHh, YYhh, YyHh, Yyhh, YyHh, Yyhh, yyHh, yyhh$

- The probability that the offspring is a dwarf plant with high yields (i.e. has genotype $yyhh$) is $1/8$. The answer is **A**.

2014 Midterm: Question 1

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- The dwarf plant has genotype $Yyhh$ and the other plant has genotype $YyHh$.
- Their offspring has possible genotypes:

$YYHh, YYhh, YyHh, Yyhh, YyHh, Yyhh, yyHh, yyhh$

- The probability that the offspring is a dwarf plant with high yields (i.e. has genotype $yyhh$) is $1/8$. The answer is **A**.
- When y and h are independent,

$$P(yyhh) = P(yy)P(hh) = 1/4 \times 1/2 = 1/8$$

2014 Midterm: Question 2

Solution: Let A_1 , A_2 , A_3 be the events that the bacteria is resistant to the antibiotic for the 1st, 2nd, 3rd patient respectively.

- $P(A_1 \cup A_2 \cup A_3) = ?$
- Note that $(A_1 \cup A_2 \cup A_3)' = A_1' \cap A_2' \cap A_3'$
- Note that A_1 , A_2 and A_3 are mutually independent.
- $P(A_1 \cup A_2 \cup A_3) = 1 - P(A_1' \cap A_2' \cap A_3') = 1 - (1 - 0.22)^3 = 0.525$.
- The answer is **D**.

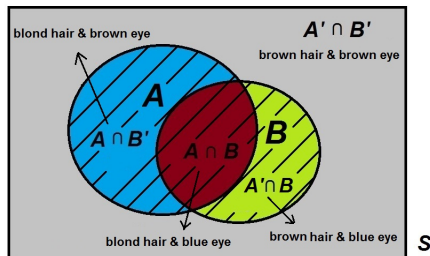
2014 Midterm: Question 3

Solution: Re-rank the data by ascending order

$$Y = \{121, 125, 134, 147, 157, 305, 315\}$$

- To compute Q_1 ($r = 1$, $n = 7$),
 - ① $(r/4)(n + 1) = (1/4)(7 + 1) = 2.00 = 2(a) + 0.00(b)$
 - ② $Q_1 = (1 - b)y_a + (b)y_{a+1} = 1 \times 125 + 0 \times 134 = 125$
- To compute Q_3 ($r = 3$, $n = 7$),
 - ① $(r/4)(n + 1) = (3/4)(7 + 1) = 6.00 = 6(a) + 0.00(b)$
 - ② $Q_3 = (1 - b)y_a + (b)y_{a+1} = 1 \times 305 + 0 \times 315 = 305$
- $IQR = Q_3 - Q_1 = 305 - 125 = 180$.
- Since $n = 7$ is odd, $Q_2 = 147$.
- Answer is **D**.

2014 Midterm: Question 4



Solution: Let $A = \{\text{blond hair}\}$ and $B = \{\text{blue eyes}\}$. Then, $A' = \{\text{brown hair}\}$ and $B' = \{\text{brown eyes}\}$.

$$P(A' \cap B') = \frac{320}{1770}, \quad P(A' \cap B) = \frac{250}{1770}, \quad P(A \cap B') = \frac{400}{1770},$$

$$\begin{aligned} P(A \cap B) &= 1 - [P(A' \cap B') + P(A' \cap B) + P(A \cap B')] \\ &= 1 - \left[\frac{320}{1770} + \frac{250}{1770} + \frac{250}{1770} \right] \\ &= 0.452 \quad (\text{Answer is } \mathbf{D}) \end{aligned}$$

2014 Midterm: Question 5

Solution:

- Sample mean:

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i = \frac{1}{5} (3.0 + 1.8 + 2.1 + 2.7 + 1.4) = 2.2$$

- Sample variance:

$$\begin{aligned} s^2 &= \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2 \\ &= \frac{1}{4} [(3 - 2.2)^2 + \dots + (1.4 - 2.2)^2] \\ &= 0.425 \end{aligned}$$

The answer is **C**.

2014 Midterm: Question 6

Solution:

Blood type	O	A	B	AB
No. of donors	8	3	3	1

Let O_1, O_2 be the events are the 1st and 2nd donors have blood type O respectively.

$$\begin{aligned}P(O_1 \cap O_2') + P(O_1' \cap O_2) &= P(O_1)P(O_2'|O_1) + P(O_1')P(O_2|O_1') \\&= \frac{8}{15} \times \frac{7}{14} + \frac{7}{15} \times \frac{8}{14} \\&= 0.5333\end{aligned}$$

The answer is **C**.

2014 Midterm: Question 6

Solution:

Blood type	O	A	B	AB
No. of donors	8	3	3	1

Let O_1, O_2 be the events are the 1st and 2nd donors have blood type O respectively.

$$\begin{aligned}P(O_1 \cap O_2') + P(O_1' \cap O_2) &= P(O_1)P(O_2'|O_1) + P(O_1')P(O_2|O_1') \\&= \frac{8}{15} \times \frac{7}{14} + \frac{7}{15} \times \frac{8}{14} \\&= 0.5333\end{aligned}$$

The answer is **C**.

- Note that $P(O_1 \cup O_2)$ is not the answer for this question!

2014 Midterm: Question 7

Solution: $X \sim N(16, 3^2)$. Let $Z = (X - 16)/3$. We have

$$\begin{aligned} P(12 < X < 20) &= P\left(\frac{12 - 16}{3} < \frac{X - 16}{3} < \frac{20 - 16}{3}\right) \\ &= P(-1.33 < Z < 1.33) \\ &= P(Z < 1.33) - P(Z < -1.33) \\ &= \Phi(1.33) - \Phi(-1.33) \\ &= 0.9082 - 0.0918 = 0.8164 \end{aligned}$$

The answer is **A**.

2014 Midterm: Question 8

Solution: We denote by C the event that the patient will develop oral cancer and L the event that the patient used Listerine.

- $P(C) = 0.05$, $P(L|C) = 0.33$, $P(L|C') = 0.27$
- By the total probability rule,

$$\begin{aligned}P(L) &= P(C)P(L|C) + P(C')P(L|C') \\ &= 0.05 \times 0.33 + (1 - 0.05) \times 0.27 = 0.273\end{aligned}$$

- By Bayes' rule,

$$P(C|L) = \frac{P(C \cap L)}{P(L)} = \frac{P(C)P(L|C)}{P(L)} = \frac{0.05 \times 0.33}{0.273} = 0.06$$

The answer is **C**.

2014 Midterm: Question 8

Solution: We denote by C the event that the patient will develop oral cancer and L the event that the patient used Listerine.

- $P(C) = 0.05$, $P(L|C) = 0.33$, $P(L|C') = 0.27$
- $P(L \cap C) \neq 0.33$, $P(L \cap C') \neq 0.27$
- By the total probability rule,

$$\begin{aligned}P(L) &= P(C)P(L|C) + P(C')P(L|C') \\ &= 0.05 \times 0.33 + (1 - 0.05) \times 0.27 = 0.273\end{aligned}$$

- By Bayes' rule,

$$P(C|L) = \frac{P(C \cap L)}{P(L)} = \frac{P(C)P(L|C)}{P(L)} = \frac{0.05 \times 0.33}{0.273} = 0.06$$

The answer is **C**.

2014 Midterm: Question 9

Solution: Let $U_+ = \{\text{hypertensive}\}$, $U_- = \{\text{normotensive}\}$. We note that

$$P(U_+) = 0.14, \quad P(T_+|U_+) = 0.78, \quad P(T_+|U_-) = 0.21$$

$$\begin{aligned} P(T_+) &= P(U_+)P(T_+|U_+) + P(U_-)P(T_+|U_-) \\ &= 0.14 \times 0.78 + (1 - 0.14) \times 0.21 = 0.2898 \end{aligned}$$

$$\begin{aligned} PPV = P(U_+|T_+) &= \frac{P(U_+ \cap T_+)}{P(T_+)} = \frac{P(U_+)P(T_+|U_+)}{P(T_+)} \\ &= \frac{0.14 \times 0.78}{0.2898} = 0.38 \end{aligned}$$

$$\begin{aligned} NPV = P(U_-|T_-) &= \frac{P(U_- \cap T_-)}{P(T_-)} = \frac{P(U_-)P(T_-|U_-)}{P(T_-)} \\ &= \frac{(1 - 0.14)(1 - 0.21)}{1 - 0.2898} = 0.96 \end{aligned}$$

The answer is **B**.

2014 Midterm: Question 10

x	30	40	60	69	74	79	84	89	100
$F(x)$	0	0.067	0.105	0.133	0.246	0.333	0.467	0.8	1

Solution:

$$\begin{aligned}P(70 \leq X \leq 79) &= P(X \leq 79) - P(X < 70) \\&= P(X \leq 79) - P(X \leq 69) \\&= 0.333 - 0.133 = 0.2\end{aligned}$$

The answer is **E**.

2014 Midterm: Question 10

x	30	40	60	69	74	79	84	89	100
$F(x)$	0	0.067	0.105	0.133	0.246	0.333	0.467	0.8	1

Solution:

$$\begin{aligned}P(70 \leq X \leq 79) &= P(X \leq 79) - P(X < 70) \\&= P(X \leq 79) - P(X \leq 69) \\&= 0.333 - 0.133 = 0.2\end{aligned}$$

The answer is **E**.

- The equality sign matters here: $P(X \leq 79) \neq P(X < 79)$.
- It is important to know the difference between CDF $F(x) = P(X \leq x)$ and PMF $f(x) = P(X = x)$.

2014 Midterm: Question 11

Solution: $X \sim \text{binomial}(20, 0.3)$.

$$\begin{aligned}P(10 \leq X < 13) &= P(X < 13) - P(X < 10) \\&= P(X \leq 12) - P(X \leq 9) \\&= P(X = 12) + P(X = 11) + P(X = 10)\end{aligned}$$

We can use one of the following R command:

```
pbinom(12,20,0.3) - pbinom(9,20,0.3)
dbinom(12,20,0.3) + dbinom(11,20,0.3) + dbinom(10,20,0.3)
```

The answer is **B**.

2014 Midterm: Question 11

Solution: $X \sim \text{binomial}(20, 0.3)$.

$$\begin{aligned}P(10 \leq X < 13) &= P(X < 13) - P(X < 10) \\&= P(X \leq 12) - P(X \leq 9) \\&= P(X = 12) + P(X = 11) + P(X = 10)\end{aligned}$$

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```

The answer is **B**.

- `dbinom(x, n, p)` computes $f(x) = P(X = x)$.
- `pbinom(x, n, p)` computes $F(x) = P(X \leq x)$.

2014 Midterm: Question 12

Solution: Let Y denote the number of children with high myopia. We know $Y \sim \text{binomial}(10, 0.03)$

$$\begin{aligned}P(Y \geq 2) &= 1 - P(Y \leq 1) = 1 - (P(Y = 0) + P(Y = 1)) \\&= 1 - (1 - 0.03)^{10} - 10(0.03)(1 - 0.03)^9 \\&= 0.0345\end{aligned}$$

The answer is **E**.

- To find the binomial coefficient $\binom{n}{k}$, you may use the nCr/nCk function of your calculator to save some time.

Good Luck!

- **Special office hours:** Tomorrow (Tuesday, Nov 01) 10:00 am - 12:00 am, 1:30 pm- 5:00 pm, at KED B-07C
- **Email:** cx3@uottawa.ca