$g'(x) = -\frac{3}{5}x^2$ let

Lab 1: Numerical Methods

Friday, September 11, 2015 11:37 AM

Root Finding
Interpretation
Numerical Differentiation and integration
Methods for IVP

Root Finding

Ie solve $f(x) = 0$ for x

7.4 Fixed point iteration

Suppose p is the root of f(x) that we seek, ie $f(P) = 0$

let g(x) be a related function such that $g(P) = P$
(ie root of f is fixed pt of g

start with a guess x_0 (for p) then
let $x_1 = g(x)$
$x_2 = g(x_1)$
$x_3 = g(x_2)$ etc.

ie iterate $x_{n+1} = g(x_n)$

ie generating a sequence

if this series converges, it must converge to p.

(ex. $g(x) = kf(x) + x$)

there is actually a simple condition that convergence: if $|g'(x)| < 1$ near P, the sequence $x_{n+1} = g(x_n)$ converges.

example $f(x) = x^3 + 5x - 5$

converges.

example $f(x) = x^3 + 5x - 5$

notice that $f(0) = -5$, $f(1) = 1$ } root in $[0,1]$

If $f(x) = x^3 + 5x - 5 = 0$

$\Rightarrow 5x = 5 - x^3$

$\Rightarrow x = \frac{5-x}{3}$

thus $g(x) = \frac{5 - x^3}{5}$ satisfies

the fixed pt condition

$(f(x) = 0 \Leftrightarrow g(x) = x)$

$$g'(x) = -\frac{3}{5}x^2$$

So $|g'(x)| = \frac{3}{5}x^2 \leq \frac{3}{5} < 1$

on $[0,1]$

$\therefore$ this will generate a convergent sequence

take $x_6 = 0.75$

$$x_1 = g(0.75) = \frac{5 - (0.75)^3}{5} = 0.915625$$

$$x_2 = g(x_1) = 0.846474$$

$$x_3 = g(x_2) = 0.878697$$

$$x_4 = g(x_3) = 0.864310$$

$$x_{15} = 0.868831$$

$$x_{16} = 0.268830 = x_{17}$$

This is the root to 6 decimal places

Let's check

$f(6.868830) \equiv 1.5 \times 10^{-7}$

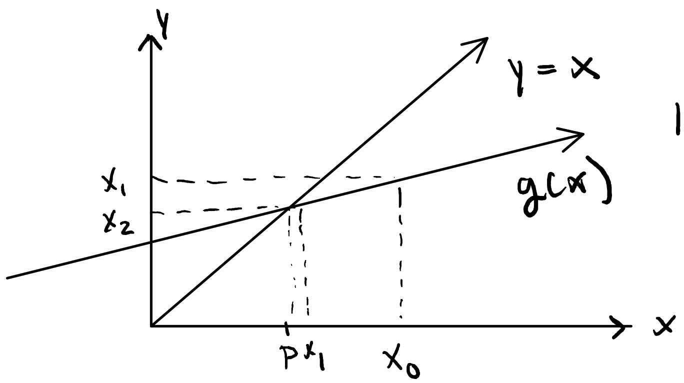
When do we stop?

3 possibilites

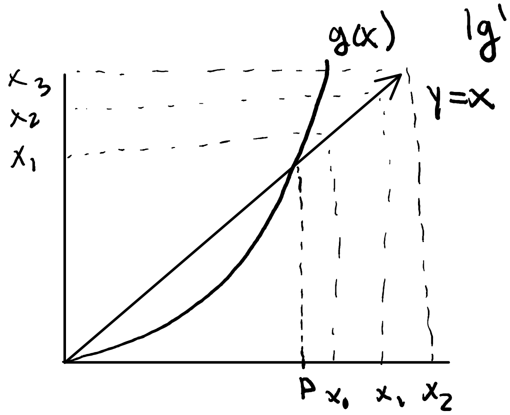
1) stop after N iteration

ii) stop after $|x_{n+1} - x_n| < \epsilon$ for tolerance ϵ (shown)

iii) stop after $|f(x_n)| < n$ for tolerance n



$|g'(x)| < 1$ near p



$|g'(x)| > 1$ near p