Office hours: MON MAR14 : 1–2 pm (KED 205G)
TUE MAR15: 1–2^{20} pm (KED B004)
no office hour on WED this week.

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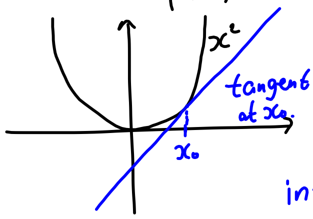
in CALC I: differentiation in one variable.
Now: functions of 2 variables

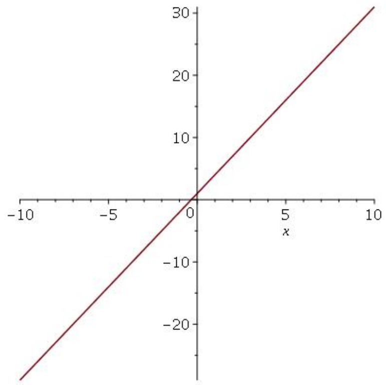
in one var: $f(x) = x^2$



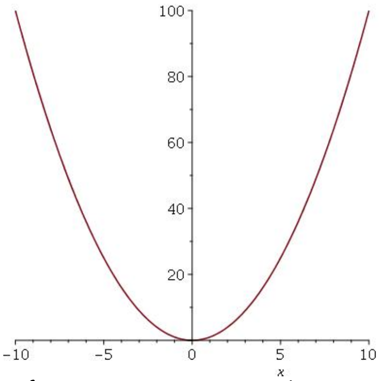
$f'(x) = 2x$
→ slope at point x
→ can compute tangent line

interpretation in one variable

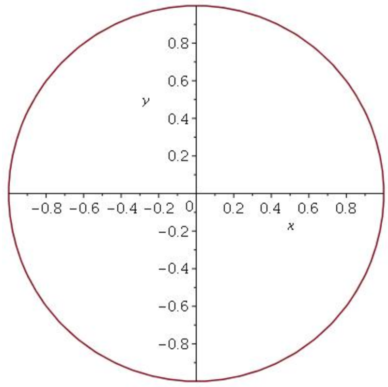
Equations in 2 variables:
(1) linear equation $y = 3x+1$



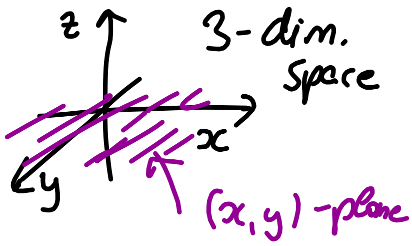
(2) $y = x^2$



(3) $x^2+y^2=1$ (circle of radius 1)
↖ equation of a curve, not a function

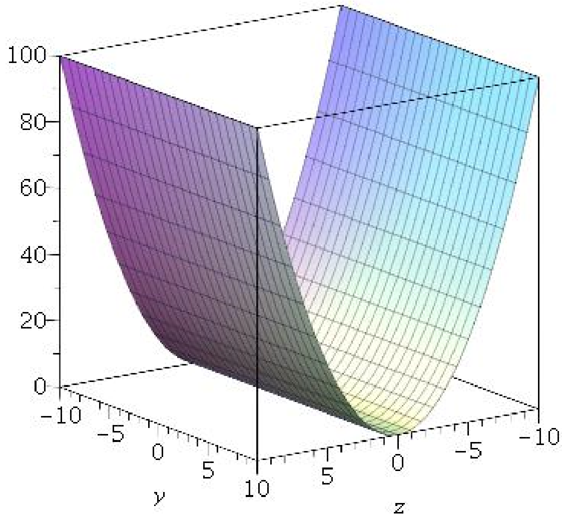


Now in 3 dimensions:



first look at previous examples:

(1) $z = x^2$ ← y not specified

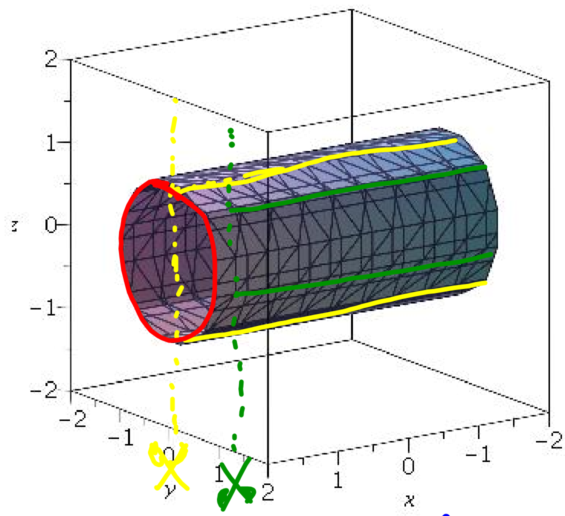


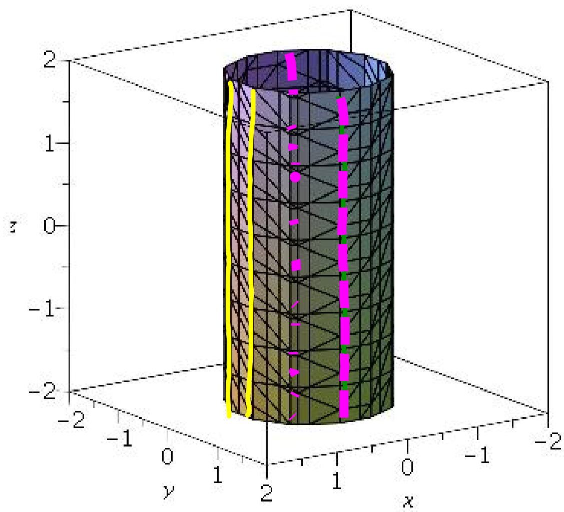
(2) in 3d: $x^2+y^2=1$ (previously ◯)

← z not specified

cylinder

(3) $y^2+z^2=1$



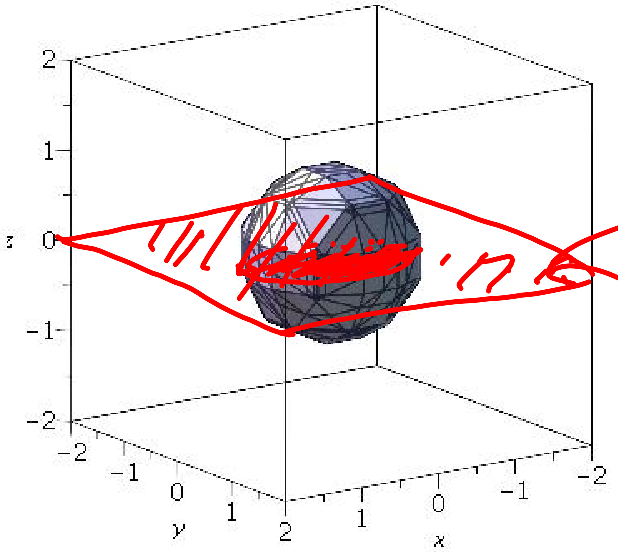


The lines are called rulings, or traces.

A cylinder has parallel rulings.

Equations in 3 variables

(1) $x^2+y^2+z^2=1$ ← sphere



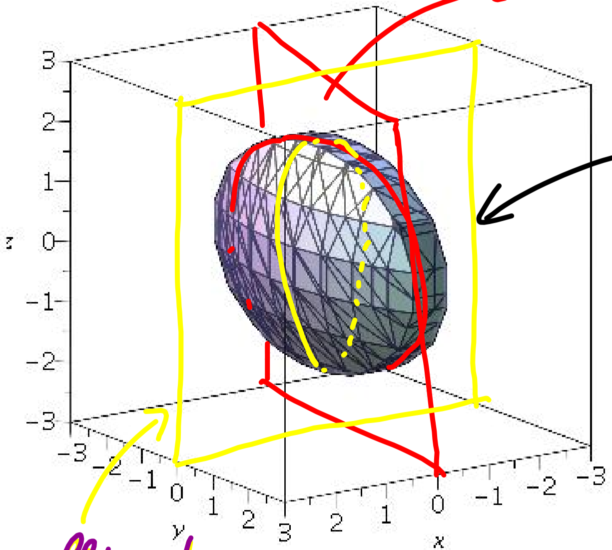
(x,y)-plane

cross section: perfect circle of radius 1

(at $z=0.5$, we get a smaller circle)

(2) $x^2+\frac{y^2}{9}+\frac{z^2}{4}=1$

cross section is an ellipse



pull/squeeze along y and z

Ellipsoid

an ellipse too

Traces are ellipses
(cross section pictures)

Try to sketch a surface:

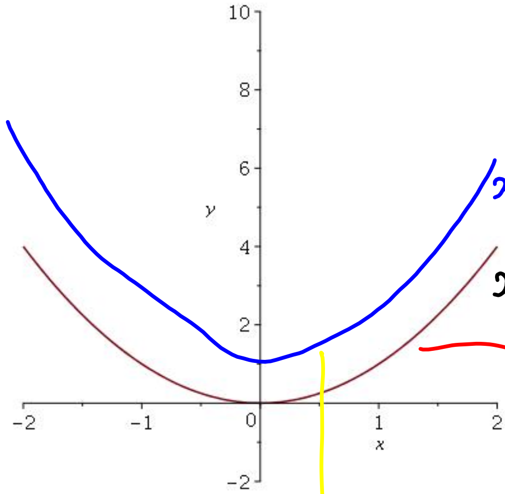
$z = 4x^2 + y^2$

(1) try sketch cross section pictures

(a) fix x: for example: $x = 0: z = 4 \cdot 0^2 + y^2 = y^2$

$x = 2: z = 4 \cdot 2^2 + y^2 = 16 + y^2$

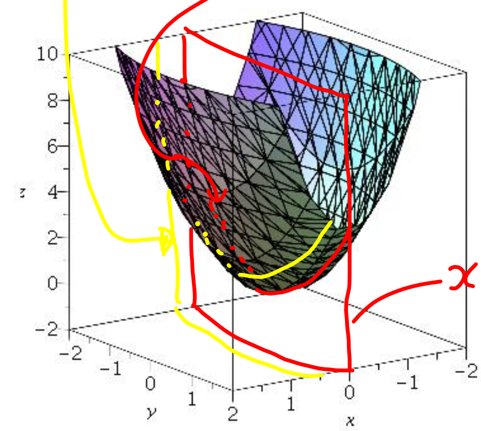
$x = \frac{1}{2}: z = 4\left(\frac{1}{2}\right)^2 + y^2 = 1 + y^2$



$x = \frac{1}{2}$

$x = 0$

final plot:

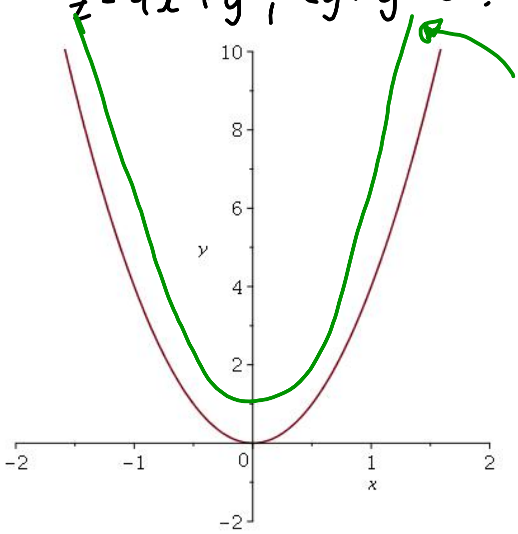


$x = 0$

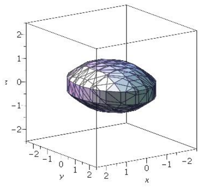
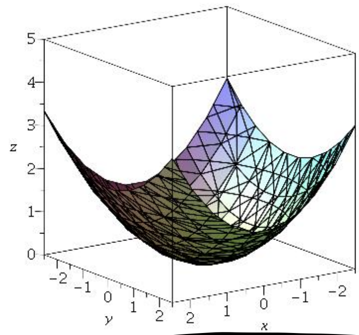
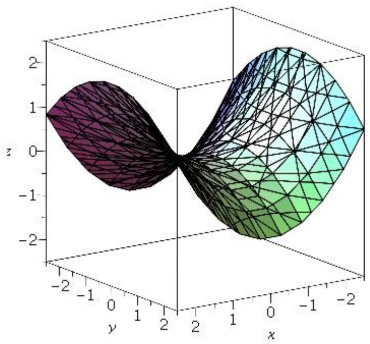
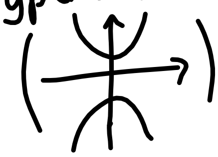
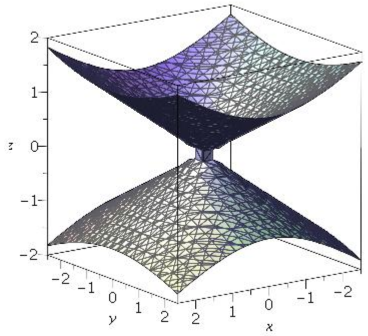
Now: look at (x,z)-plane, (= leave y fixed)

$z = 4x^2 + y^2$, eg: $y = 0$: $z = 4x^2$

$z = 4x^2 + 1$ .....



Important examples (surfaces with names):

name of surface	typical equation	traces
Ellipsoid 	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	traces are ellipses
Elliptic paraboloid 	$\frac{z}{c}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	horizontal: ellipses vertical: parabolas
Hyperbolic paraboloid: 	$\frac{z}{c}=\frac{x^2}{a^2}-\frac{y^2}{b^2}$	horizontally: hyperbolas  vertically: parabolas
Cone 	$\frac{z^2}{c^2}=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	horizontal: ellipses vertical: hyperbolas (or lines if y or $x=0$)

Tutorial for SMART Response 2013.notebook