COMP2805: Solution to assignment # 2

October 27, 2010

Question 1

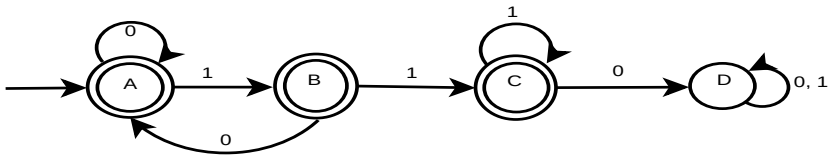
Let $\Sigma = \{a, b\}$. Write a regular expression for the set of all strings in Σ^* that start with an odd number of b's, and contain at least two a's.

$b(bb)^*ab^*a(a+b)^*$

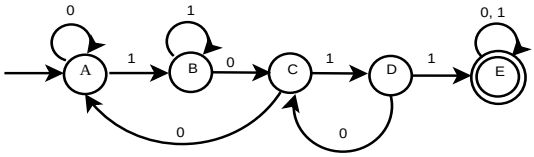
Question 2

For each of the following languages, construct a DFA that accepts the language. In all cases, the alphabet is {0, 1}.

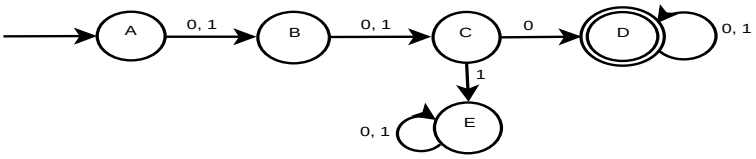
2.1 part 2) $\{w : 110$ is not a substring of $w\}$



2.1 part 4) $\{w : w$ contains the substring $1011\}$

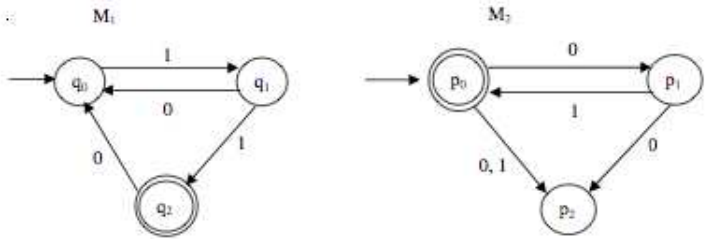


2.1 part 9) $\{w : w$ has length at least 3 and its third symbol is $0\}$



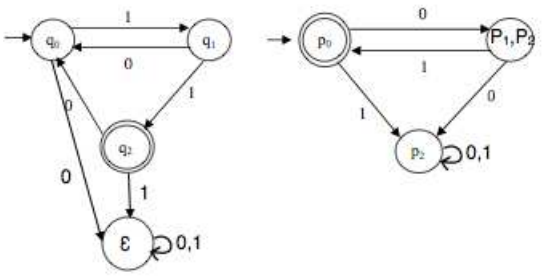
Question 3

Problem 2.2 (except Part 2) from the Lecture Notes (Page 75), in which the two NFAs M1 and M2 are:



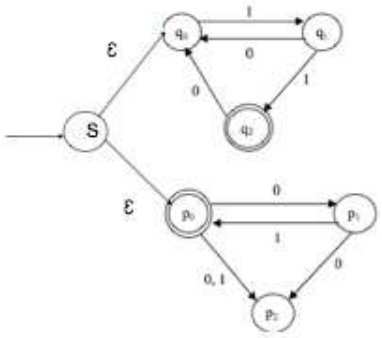
2.2 Part 1)
M1 and M2 are NFAs because both M1 and M2 have at lease one transition function with an output that is not to a unique state. For example M1 $F(q_0, 0) = \phi$, empty set, and M2 $F(p_0, 0) = \{p_1, p_2\}$.

2.2 Part 3)

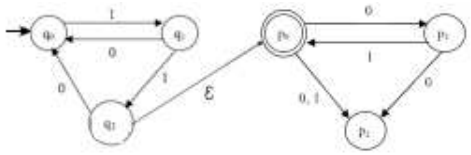


There is a " typo" in the figure on the left. The ϵ used as a state index should rather be read as Φ.

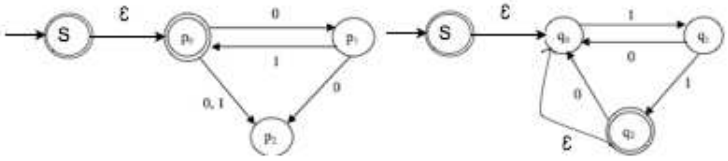
2.2 Part 4)



2.2 Part 5)



2.2 Part 6)



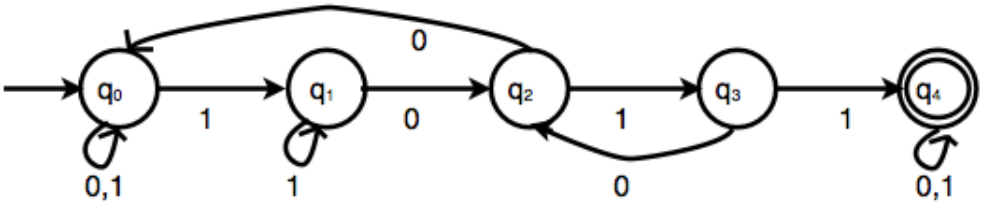
2.2 Part 7)

(10 +110)*11

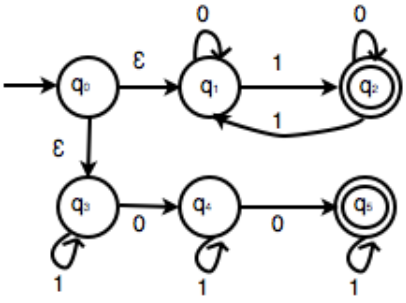
Question 4

Problem 2.3 from the Lecture Notes (Page 75): Parts 2 and 3.

2.3 Part 2)

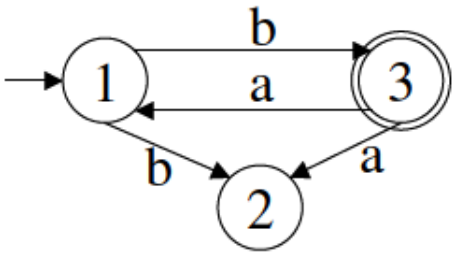


2.3 Part 3)

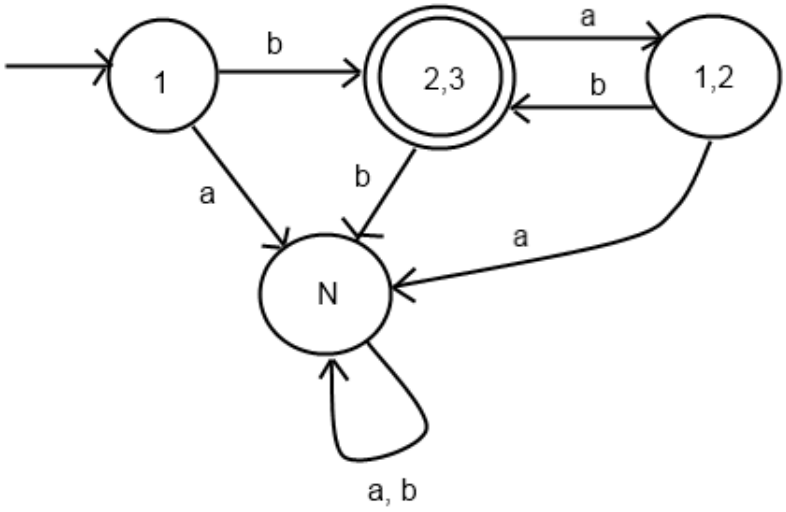


Question 5

Convert the following NFA into an equivalent DFA and give the language they recognize.



DFA (Simplified)



Language for this NFA/DFA: $L = b\{ab\}^*$

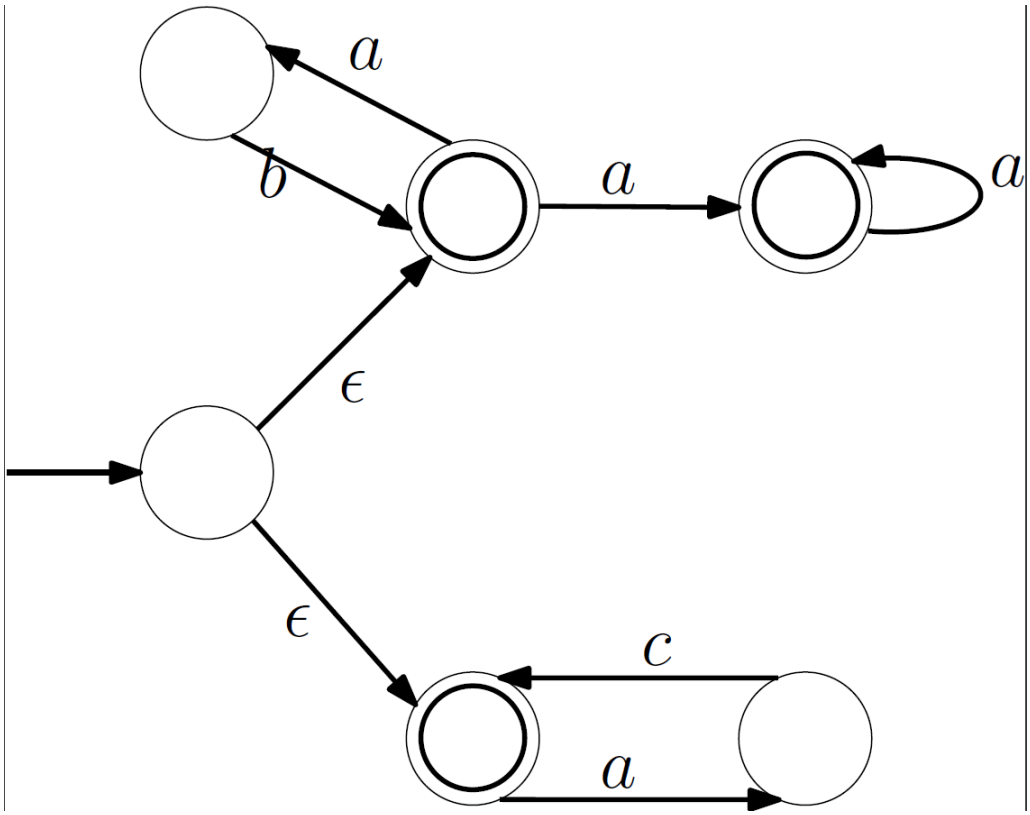
Question 6

Give regular expressions describing the following languages in which the alphabet S is {0,1}.
a. {W| W has at least two 1s}
b. {W| W has at most two 1s}
c. {W| W has exactly two 1s}

6a) $L = \{0|1\}^*1\{0|1\}^*1\{0|1\}^*$
6b) $L = 0^*\{0|1\}0^*\{0|1\}0^*$
6c) $L = 0^*10^*10^*$

Question 7

Develop the NFA for the regular expression $(ab)^*a^* \cup (ac)^*$.

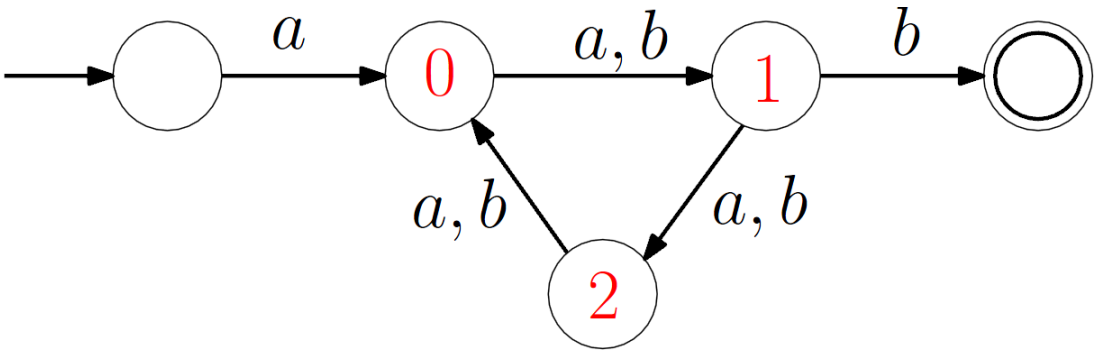


Question 8

Let $L = \{awb | w \in \{a, b\}^* \wedge |w| = 3n + 1, n \in N\}$ be a language over alphabet $\Sigma = \{a, b\}$. Decide whether or not L is a regular language and provide a proof to justify your answer.

The proof that a language is regular is to provide an NFA or a DFA that recognise the language. If the language is not regular, then the proof is done with the pumping lemma.

Language L is regular.



States with red numbers are the w part of the language. As describe above, the length of $|w| = 3n + 1$. Another way to see this is that when $|w|$ is divided by 3, the remainder is 1[1]. The states with red numbers identify the remainder of the length of w.

[1]With $3n$, the remainder would be 0 and with $3n + 2$, the remainder would be 2