WORK (§6.4)

Newton's 2nd law of motion:

(1) $$F = m \cdot \frac{d^2 s}{dt^2}$$

force, mass, 2nd derivative of displacement (accelleration)

units: $N = \frac{kg\,m}{s^2}$, kg, $\frac{m}{s^2}$

(2) $$\boxed{W = F \cdot s}$$ $(= m \cdot g \cdot s$ if accell. $= g)$

work, force, displacement

$J = N{\cdot}m$, N, m

Joule

Ex: How much work is done by climbing a 350m hill?

Cyclist: 55 kg, bike: 11 kg, water & tools: 2 kg

(need mass for force and force for work!)

accelleration here: gravity $\boxed{g = 9.81 \frac{m}{s^2}}$

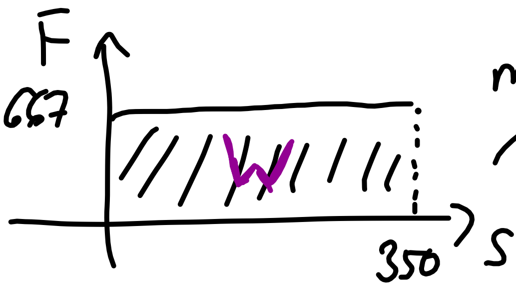
unit at end

$$F = m \cdot g = (55+11+2) \cdot 9.81 = 667.08\,N$$

$$W = F \cdot s = 667.08 \cdot 350m = 233478\,J$$

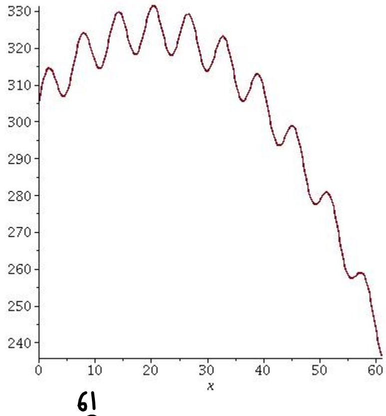
in general: force not constant!

→ use integrals!



more realistic →

We approximate the force used to climb by the function for the path $f(x) = \left(-\left(\frac{x-20}{10}\right)^2 + 65\right)\cdot\left(\frac{\sin(x)}{10} + 5\right)$



f(x) describing path!

use formula

$W = m\cdot g\cdot s$

and compute s via integral!

↖ describes altitude at km x!
(it's 61 km long)

$$W = \int_0^{61} m\cdot g\cdot f(x)\,dx$$

$$= \int_0^{61} m\cdot g\cdot\left(\left(-\frac{x-20}{10}\right)^2 + 65\right)\cdot\left(\frac{\sin(x)}{10} + 5\right)dx$$

practice: multiply out!

$$= \int_0^{61} mg\cdot\left(-\frac{x^2\cdot\sin(x)}{1000} + \frac{4}{100}x\cdot\sin(x) + \frac{4}{10}\sin(x) - \frac{x^2}{20} + 2x + 305\right)dx$$

$$= mg\int_0^{61} \ldots$$ split up into parts!

int. by parts (exercise - ask TA!!)

$$\frac{1}{1600}\int_0^{61} x^2\cdot\sin(x)\,dx = 900\cdot 20\cdot\frac{1}{1000} = 0.9002$$

int. by parts

$$\frac{4}{100}\int_0^{61} x\cdot\sin(x)\,dx = 0.5908$$

$$\frac{4}{10}\int_0^{61}\sin(x)\,dx = 0.503$$

$$\int_0^{61}\ldots = 17017.983$$

$$W = m\cdot g\cdot\int_0^{61} f(x)\,dx = m\cdot g\cdot(0.9002 + 0.5908 + 0.503 + 17017.983$$

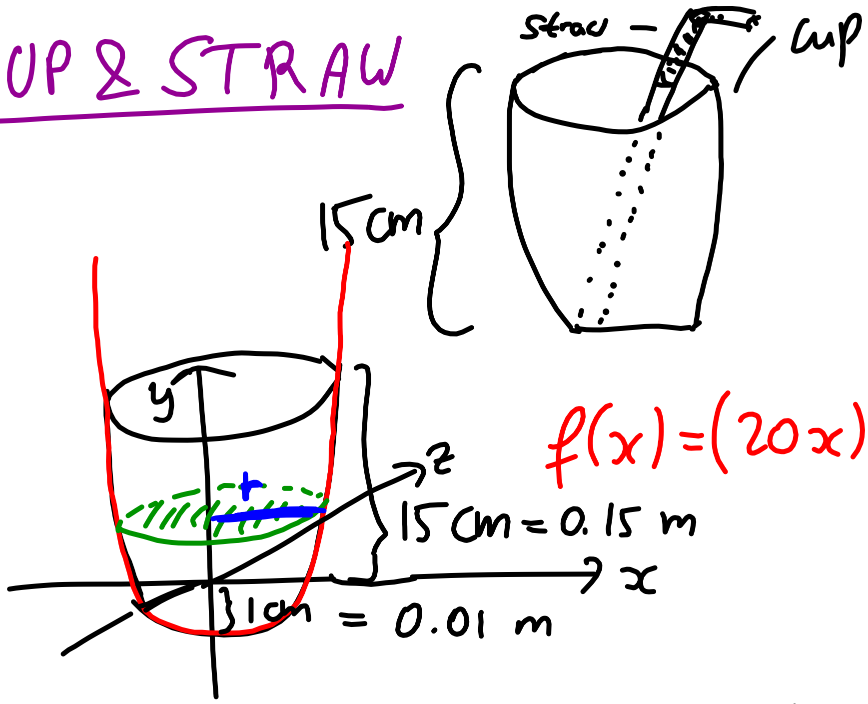
$$= 17020.646\; m\cdot g$$

$m = 55 + 11 + 2$ kg
$g = 9.81\,\frac{m}{s^2}$

$$= 11354101.853$$

this is more work than just climbing!

CUP & STRAW



$f(x) = (20x)^2 - 1$

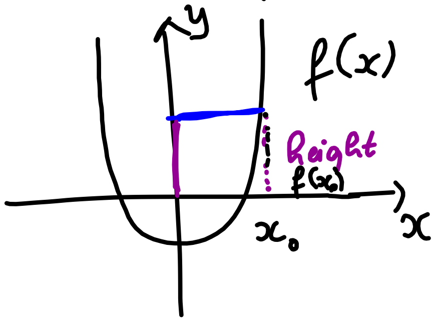
last week: volume of rotational objects!

again: need cross section area function!
(area of it depends on height!)

Cross section area is a disc (solid circle): area depending on radius r: $A = r^2 \cdot \pi$. So here: find r depending on height!

We integrate over y, so first: find $y = (20x)^2 - 1$
express x separately!

How do we find r? – depending on height



height at x is just function value!!

reverse f to find r!

$$y = (20x)^2 - 1 \quad (= f(x))$$

$$\rightsquigarrow x = \frac{\sqrt{y+1}}{20} \quad \text{(exercise) (ask TA or me)}$$

We get cross section radius:

$$r(y) = \frac{\sqrt{y+1}}{20}$$

↑ height

Want: work it take to pump up water with a straw from

(a) bottom of the cup

(b) from liquid surface

$$\underset{\text{mass}}{m} = \underset{\text{volume}}{V} \cdot \underset{\text{density}}{d}$$

recall: $W = F \cdot s = m \cdot g \cdot s = V \cdot d \cdot g \cdot s$

Here: s is height of cup $= 0.15$ m

g: $g = 9.81 \frac{m}{s^2}$

d: density of water: weight per m^3; 1000 kg/m^3

need volume! via integral.

radius of cross section was $r(y) = \frac{\sqrt{y+1}}{20}$

So AREA: $A(y) = (r(y))^2 \cdot \pi = \frac{y+1}{400} \cdot \pi$

$$V = \int_0^{0.15} A(y)\,dy = \frac{\pi}{400}\int_0^{0.15} y+1\,dy = \frac{\pi}{400}\left[\frac{y^2}{2} + y\right]_0^{0.15}$$

$$= \underline{\underline{0.001266455 \text{ m}^3}} \approx \underline{\underline{1.266 \ \ell}}$$

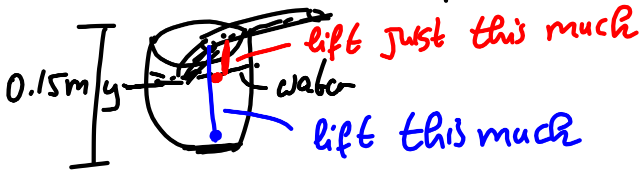
makes sense!!

in (a), pump from bottom. All the water has to be lifted exactly 0.15 m (15 cm).

$$W = V \cdot d \cdot g \cdot s = 0.00126655\ m^3 \cdot 1000\ \frac{kg}{m^3} \cdot 9.81\ \frac{m}{s^2} \cdot 0.15\ m$$

$$= \underline{1.8636\ J}$$ (did not use integral for work!)

in (b): take water from top:



not same distance!

We lift a molecule (H_2O) at height y up to 0.15 m, so we lift it $(0.15 - y)$ m.

$$W = d \cdot g \cdot \int_0^{0.15} \underbrace{\underbrace{(y+1)}_{\text{radius square}} \cdot \frac{\pi}{400}}_{\text{cross sect. area}} \cdot \underbrace{(0.15 - y)}_{\text{displacement}}\, dy$$

$$= 1000\ \frac{kg}{m^2} \cdot 9.81\ \frac{m}{s^2} \cdot \frac{\pi}{400} \cdot \int_0^{0.15} 0.15y - y^2 + 0.15 - y\, dy$$

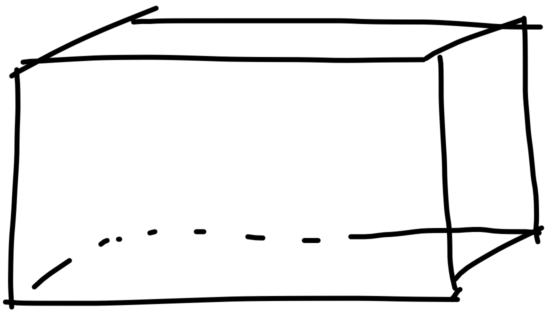
$$= \text{—} \cdot \text{—} \cdot \left[0.15\frac{y^2}{2} - \frac{y^3}{3} + 0.15y - \frac{y^2}{2}\right]_0^{0.15}$$

$$= \underline{0.91013\ J}$$

(a) gave 1.86 J, in (b) we only need 0.91 J.
It takes less energy to pump from the top.

For a cup, about 10^6 liters makes the energy of one cookie. Doesn't matter.

BUT: have a tank, want to empty the water with a pump.



It takes a lot more energy to pump from the bottom! So: use a floating pump to preserve energy!!