Total mark: 40. Closed book. Non-programmable calculators are allowed.

Last Name ____________ ***First Name*** ____________ ***Student Number*** ____________

Question 1. [6 Marks] Multiple Choice Questions(Please **CIRCLE** your answer)

(1) (2 Marks) Which of the following sets of vectors are linearly **dependent**?

$$A = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 40 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\} \qquad B = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$C = \left\{ \begin{bmatrix} 4 \\ 4 \\ 4 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} \right\} \qquad D = \left\{ \begin{bmatrix} 7 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 100 \\ -200 \\ 300 \end{bmatrix} \right\}$$

A. A, B, C, D B. B, C, D only C. A, B, D only D. A, B, C only

(2) (2 Marks) Which of the following sets of vectors spans $\mathbb{R}^3$?

$$A = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\} \qquad B = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$$

$$C = \left\{ \begin{bmatrix} 1 \\ 20 \\ 30 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} \quad D = \left\{ \begin{bmatrix} 3 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \end{bmatrix} \right\}$$

A. A, B, C only **B.** A, C **only** C. B, C only D. B, D only

(3) (2 Marks) Consider two matrices A, B, C. If A is a 3×2 matrix, C is a 2×4 matrix and ABC can be computed, what must be the dimensions of B

A. 3×3 **B.** 2×2 C. 3×4 D. 2×4

Solution: (1) A (2) B (3) B

Question 2. [10 Marks] Let

$$A = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 3 & 3 \end{pmatrix}, \; B = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 3 \\ 2 & 3 \\ 3 & 3 \end{pmatrix}, \; D = \begin{pmatrix} 1 & 0.5 \\ 0 & 1 \end{pmatrix}$$

1) [3 Marks]Find $A - C'$.
2) [2 Marks]Find B^4.
3) [3 Marks]Find BA.
4) [2 Marks]Find BD^T.

Solution:

$$A - C' = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$B^4 = \begin{pmatrix} 1 & 0 \\ -8 & 1 \end{pmatrix}$$

$$BA = \begin{pmatrix} 2 & 3 & 4 \\ -1 & -3 & -5 \end{pmatrix}$$

$$BD' = \begin{pmatrix} 1 & 0 \\ -1.5 & 1 \end{pmatrix}$$

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Question 3. [24 Marks] Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & -3 \\ -1 & -4 & -4 \end{bmatrix}$$

and the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} \quad \mathbf{v}_2 = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix} \quad \mathbf{v}_3 = \begin{bmatrix} 1 \\ -3 \\ -4 \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$$

($\mathbf{v}_i (i = 1, 2, 3)$ is the ith column of A). Let $W = \text{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$

1) [6 Marks] Solve the matrix equation $A\,\mathbf{x} = \mathbf{u}$.

Solution: The augmented matrix[1 Marks] is

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 1 & 0 & -3 & 2 \\ -1 & -4 & -4 & 0 \end{array}\right]$$

(1 point for each correct row operation, maximum 3 Marks). The solution[3 Marks] is

$$x_1 = -4,\ x_2 = 3,\ x_3 = -2$$

2) [2 Marks] Are the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ linearly independent? Why or why not?

Solution: The vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly independent[1 Mark], since the row rank is 3[1 Mark].

3) [2 Marks] Are the vectors $\mathbf{v}_1$, $\mathbf{v}_3$ linearly independent? Why or why not?

Solution: The vectors $\mathbf{v}_1, \mathbf{v}_3$ are linearly independent[1 Mark], since the row rank is 2[1 Mark] or they are subset of a independent set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$.

4) [3 Marks] Determine if it is possible to express the vector $\mathbf{u}$ as a linear combination of the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. If it is possible, write $\mathbf{u}$ explicitly as a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

Solution:

$$\mathbf{u} = -4\,\mathbf{v}_1 + 3\,\mathbf{v}_2 - 2\,\mathbf{v}_3$$

5) [2 Marks] Are the vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{u}$ linearly independent or dependent? why ?

Solution: Since

$$\mathbf{u} = -4\,\mathbf{v}_1 + 3\,\mathbf{v}_2 - 2\,\mathbf{v}_3$$

they are linearly dependent. Or since the rank is 3 or this set has 4 three dimensional vectors.

6) [3 Marks] Find the solution to the homogeneous system $A\,\mathbf{x} = \mathbf{0}$.

Solution: Since the row rank of A is 3, this homogeneous system has only $\mathbf{0}$ solution which means $x_1 = 0$, $x_2 = 0$, $x_3 = 0$.

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7) [1 Mark] Is $\mathbf{u}$ in W, why?

Solution: Yes, since $A\,\mathbf{x} = \mathbf{u}$ is consistent.

8) [1 Mark] Is $\mathbf{v}_1$ in W?

Solution: Yes.

9) [2 Marks] Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbb{R}^3$, why?

Solution: Yes. Since the row rank of A is 3.

10) [2 Marks] Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{u}\}$ span $\mathbb{R}^3$, why?

Solution: Yes. Since the row rank of A is 3.