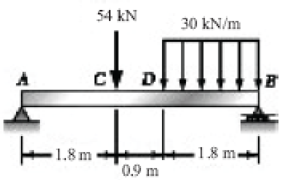
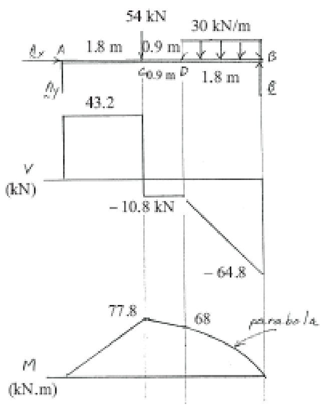
PROBLEM 7.38



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



(*a*) **FBD Beam:**

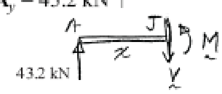
$$\Sigma M_A = (4.5\text{ m})B - (3.6\text{ m})(30\text{ kN/m})(1.8\text{ m}) - (1.8\text{ m})(54\text{ kN}) = 0$$

$$\mathbf{B} = 64.8\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y\,(-54\text{ kN}) - (30\text{ kN/m})(1.8\text{ m}) + 64.8\text{ kN}$$

$$\mathbf{A}_y = 43.2\text{ kN} \uparrow$$

Along AC:



$$\uparrow \Sigma F_y = 0: 43.2\text{ kN} - V = 0$$

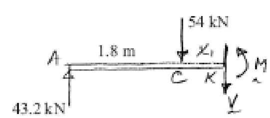
$$V = 43.2\text{ kN}$$

$$\Sigma M_J = 0: M - x(43.2\text{ kN}) = 0$$

$$M = (43.2\text{ kN})\,x$$

$$M = 77.8\text{ kN.m at } C\ (x = 1.8\text{ m})$$

Along CD:



$$\uparrow \Sigma F_y = 0: 43.2\text{ kN} - 54\text{ kN} - V = 0$$

$$V = -10.8\text{ kN}$$

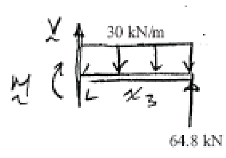
$$\Sigma M_K = 0: M + x_1\,(54\text{ kN}) - (1.8\text{ m} + x_1)(43.2\text{ kN}) = 0$$

$$M = 77.8\text{ kN.m} - (10.8\text{ kN})\,x_1$$

$$M = 68\text{ kN.m at } D$$

PROBLEM 7.38 CONTINUED

Along DB:



$$\Sigma F_y = 0: V - x_3\,(30\text{ kN/m}) + 64.8\text{ kN} = 0$$

$$V = -64.8\text{ kN} + (30\text{ kN/m})\,x_3$$

$$V = -10.8\text{ kN at } D$$

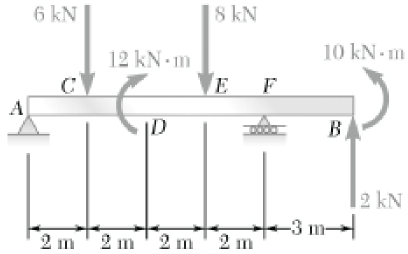
$$\Sigma M_L = 0: M + \frac{x_3}{2}\,(30\text{ kN/m})(x_3) - x_3\,(64.8\text{ kN}) = 0$$

$$M = (64.8\text{ kN})x_3 - (15\text{ kN/m})x_3^2$$

$$M = 68\text{ kN at } D\ (x_3 = 1.8\text{ m})$$

(*b*) From diagrams: $|V|_{max} = 64.8\text{ kN at } B$ ◀

$|M|_{max} = 77.8\text{ kN.m at } C$ ◀

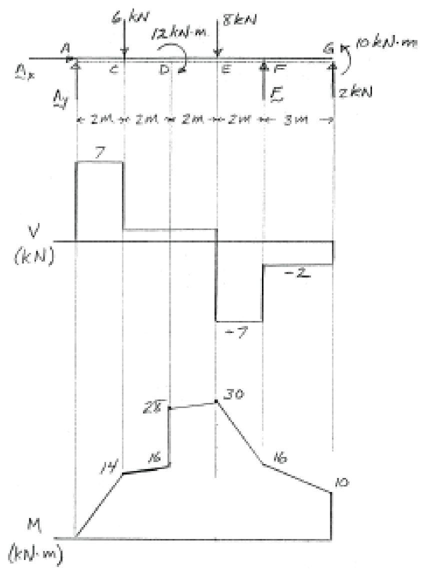


PROBLEM 7.71

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*)



FBD Beam:

$$\circlearrowleft \Sigma M_A = 0: (8\text{ m})F + (11\text{ m})(2\text{ kN}) + 10\text{ kN·m} - (6\text{ m})(8\text{ kN})$$

$$-12\text{ kN·m} - (2\text{ m})(6\text{ kN}) = 0 \quad \mathbf{F} = 5\text{ kN} \uparrow$$

$$\uparrow \Sigma F_y = 0: A_y - 6\text{ kN} - 8\text{ kN} + 5\text{ kN} + 2\text{ kN} = 0$$

$$\mathbf{A}_y = 7\text{ kN} \uparrow$$

Shear Diag:

V is piecewise constant with discontinuities equal to the concentrated forces at A, C, E, F, G

Moment Diag:

M is piecewise linear with a discontinuity equal to the couple at D.

$$M_C = (7\text{ kN})(2\text{ m}) = 14\text{ kN·m}$$

$$M_{D^-} = 14\text{ kN·m} + (1\text{ kN})(2\text{ m}) = 16\text{ kN·m}$$

$$M_{D^+} = 16\text{ kN·m} + 12\text{ kN·m} = 28\text{ kN·m}$$

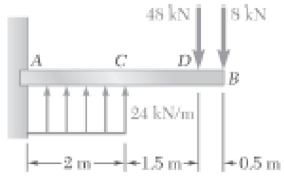
$$M_E = 28\text{ kN·m} + (1\text{ kN})(2\text{ m}) = 30\text{ kN·m}$$

$$M_F = 30\text{ kN·m} - (7\text{ kN})(2\text{ m}) = 16\text{ kN·m}$$

$$M_G = 16\text{ kN·m} - (2\text{ kN})(3\text{ m}) = 10\text{ kN·m}$$

(*b*) $|V|_{max} = 7.00\text{ kN}$ ◀

$|M|_{max} = 30.0\text{ kN}$ ◀



PROBLEM 7.38

For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION

(*a*) **FBD Beam:**

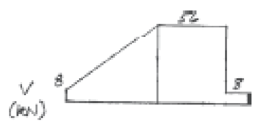
$\leftarrow \Sigma F_x = 0: \quad A_x = 0$

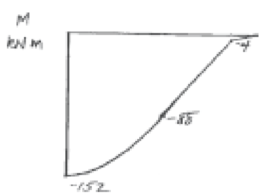
$\uparrow \Sigma F_y = 0: \quad A_y + (2\text{ m})(24\text{ kN/m}) - 48\text{ kN} - 8\text{ kN} = 0$

$$\mathbf{A}_y = 8\text{ kN}\uparrow$$

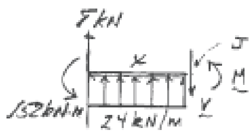
$\circlearrowleft \Sigma M_A = 0: \quad M_A + (1\text{ m})(2\text{ m})(24\text{ kN/m}) - (3.5\text{ m})(48\text{ kN})$

$$-(2\text{ m})(8\text{ kN}) = 0, \qquad \mathbf{M}_A = 152\text{ kN}\cdot\text{m}$$





Along AC:



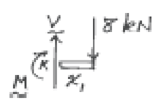
$\uparrow \Sigma F_y = 0: \quad 8\text{ kN} + x(24\text{ kN}\cdot\text{m}) - V = 0$

$$V = 8\text{ kN} + (24\text{ kN/m})x$$

$\circlearrowleft \Sigma M_J = 0: \quad M + 152\text{ kN}\cdot\text{m} - x(8\text{ kN}) - \frac{x}{2}(24\text{ kN/m})x = 0$

$$M = (12\text{ kN/m})x^2 + (8\text{ kN})x - 152\text{ kN}\cdot\text{m}$$

Along DB:



$\uparrow \Sigma F_y = 0: \quad V - 8\text{ kN} = 0$

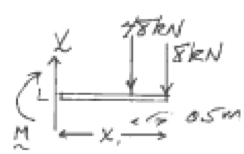
$$V = 8\text{ kN}$$

$\circlearrowright \Sigma M_K = 0: \quad M + x_1(8\text{ kN}) = 0, \qquad M = -(8\text{ kN})x_1$

continued

PROBLEM 7.38 CONTINUED

Along CD:



$\uparrow \Sigma F_y = 0: \quad V - 48\text{ kN} - 8\text{ kN} = 0, \quad V = 56\text{ kN}$

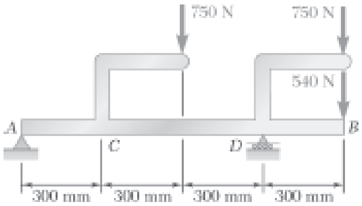
$\circlearrowright \Sigma M_L = 0: \quad M + (x_1 - 0.5\text{ m})(48\text{ kN}) + x_1(8\text{ kN}) = 0$

$$M = 24\text{ kN}\cdot\text{m} - (56\text{ kN})x_1$$

(*b*)

$$|V|_{max} = 56.0\text{ kN along } CD \blacktriangleleft$$

$$|M|_{max} = 152.0\text{ kN}\cdot\text{m at } A \blacktriangleleft$$

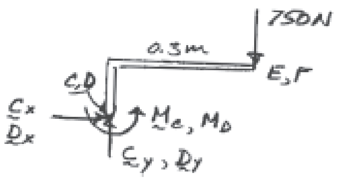


PROBLEM 7.46

Draw the shear and bending-moment diagrams for the beam AB, and determine the maximum absolute values of the shear and bending moment.

SOLUTION

FBD CE or DF:



$\rightarrow \Sigma F_x = 0$: C_x, $D_x = 0$

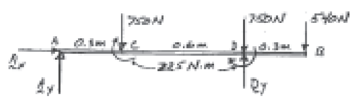
$\uparrow \Sigma F_y = 0$: $C_y - 750\text{ N} = 0$, $\quad C_y = 750\text{ N}$

$D_y = 750\text{ N}$

$\circlearrowleft \Sigma M_C = 0$: $M_C - (0.3\text{ m})(750\text{ N}) = 0$

$M_C = 225\text{ N}\cdot\text{m} = M_D$

Beam AB:



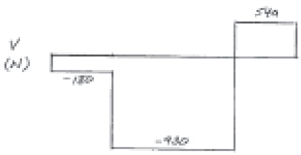
$\circlearrowleft \Sigma M_A = 0$: $(0.9\text{ m})D_y - 2(225\text{ N}\cdot\text{m}) - (0.3\text{ m})(750\text{ N})$

$-(0.9\text{ m})(750\text{ N}) - (1.2\text{ m})(540\text{ N}) = 0$

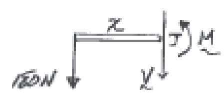
$D_y = 2220\text{ N}$

$\uparrow \Sigma F_y = 0$: $A_y - 2(750\text{ N}) - 540\text{ N} + 2220\text{ N} = 0$

$A_y = -180\text{ N} \qquad \mathbf{A}_y = 180\text{ N}\downarrow$



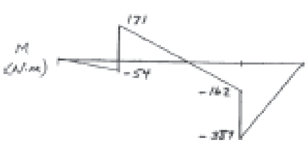
Along AC:



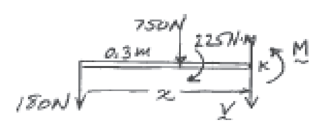
$\uparrow \Sigma F_y = 0$: $-180\text{ N} - V = 0$

$V = -180\text{ N}$

$\circlearrowleft \Sigma M_J = 0$: $M + x(180\text{ N}) = 0 \qquad M = (180\text{ N})x$



Along CD:



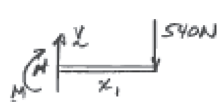
$\uparrow \Sigma F_y = 0$: $-180\text{ N} - 750\text{ N} - V = 0, \qquad V = -930\text{ N}$

$\circlearrowleft \Sigma M_K = 0$: $M - 225\text{ N}\cdot\text{m} + (x - 0.3\text{ m})(750\text{ N}) + x(180\text{ N}) = 0$

$M = 450\text{ N}\cdot\text{m} - (930\text{ N})x$

PROBLEM 7.46 CONTINUED

Along DB:



$\uparrow \Sigma F_y = 0$: $V - 540\text{ N} = 0$

$V = 540\text{ N}$

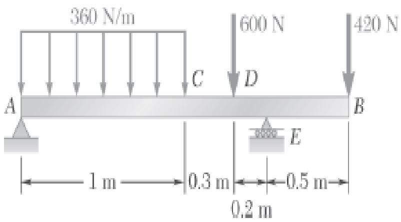
$\circlearrowright \Sigma M_N = 0$: $M + x_1(540\text{ N}) = 0 \qquad M = -(540\text{ N})x_1$

Note: The discontinuities in M, at C and D, equal 225 N·m, M_C and M_D

From the diagrams $\qquad |V|_{max} = 930\text{ N}$ along CD ◀

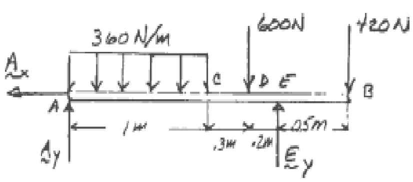
$|M|_{max} = 387\text{ N}\cdot\text{m}$ at D ◀

PROBLEM 7.64



For the beam and loading shown, (*a*) draw the shear and bending-moment diagrams, (*b*) determine the maximum absolute values of the shear and bending moment.

SOLUTION



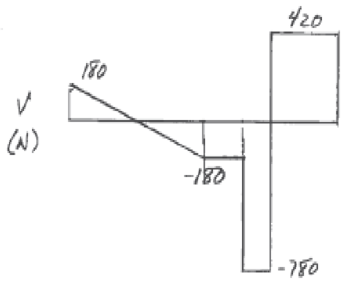
$$\circlearrowleft \Sigma M_A = 0: \quad (1.5\text{ m})E_y - \left(\frac{1}{2}\text{m}\right)(360\text{ N/m})(1\text{ m}) - (1.3\text{ m})(600\text{ N})$$

$$-(2\text{ m})(420\text{ N}) = 0 \qquad \mathbf{E}_y = 1200\text{ N}\uparrow$$

$$\uparrow \Sigma F_y = 0: \quad A_y + 1200\text{ N} - (360\text{ N/m})(1\text{ m}) - 600\text{ N} - 420\text{ N} = 0$$

(*a*)

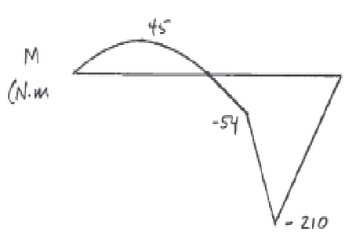
$$\mathbf{A}_y = 180.0\text{ N}\uparrow$$



Shear Diag:

V jumps to $A_y = 180$ N at A, then decreases linearly $\left(\dfrac{dV}{dx} = -w = -360\text{ N/m}\right)$ to $180\text{ N} - (360\text{ N/m})(1\text{ m}) = -180\text{ N}$ at C.

From C, V is piecewise constant $(w = 0)$ with jumps of -600 N at D, $+1200$ N at E, -420 N at B.

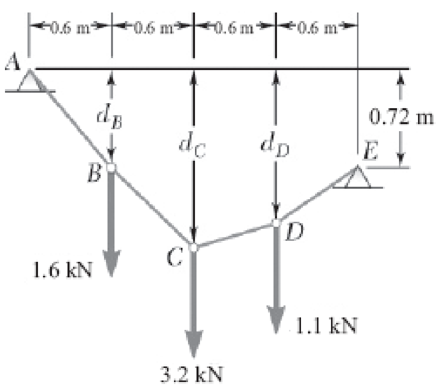


Moment Diag:

M starts at zero at A with slope $\dfrac{dM}{dx} = V = 180$ N/m, decreasing to zero at $x = 0.5$ m. There $M = \dfrac{1}{2}(180\text{ N})(0.5\text{ m}) = 45\text{ N}\cdot\text{m}$. M is zero again at C, decreasing to $-(180\text{ N})(0.3\text{ m}) = -54\text{ N}\cdot\text{m}$ at D. M then decreases by $(780\text{ N})(0.2\text{ m}) = 156\text{ N}\cdot\text{m}$ to $-210\text{ N}\cdot\text{m}$ at E, and increases by $(420\text{ N})(0.5\text{ m}) = 210\text{ N}\cdot\text{m}$ to zero at B.

(*b*) From the diagrams, $|V|_{max} = 780$ N along EB ◀

$|M|_{max} = 210\text{ N}\cdot\text{m}$ at E ◀

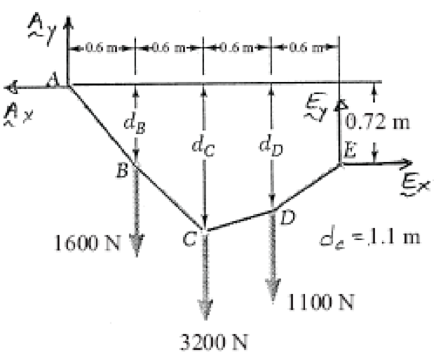


PROBLEM 7.92

Cable *ABCDE* supports three loads as shown. Knowing that $d_C = 1.1$ m, determine (*a*) the reaction at *E*, (*b*) the distances d_B and d_D.

SOLUTION

FBD Cable:



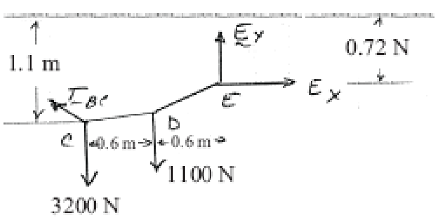
(*a*) $\circlearrowleft \Sigma M_A = 0$: $(0.72\text{ m})\,E_x + (2.4\text{ m})\,E_y - (0.6\text{ m})\,(1600)$

$$-(1.2\text{ m})\,(3200\text{ N}) - (1.8\text{ m})\,(1100\text{ N}) = 0$$

$$0.3\,E_x + E_y = 2825\text{ N} \qquad (1)$$

$\circlearrowleft \Sigma M_C = 0$: $-(0.38\text{ m})\;E_x + (1.2\text{ m})\,E_y - (0.6\text{ m})\,(1100\text{ N}) = 0$

FBD CDE:



$$-0.3167\,E_x + E_y = +550\text{ N} \qquad (2)$$

Solving (1) and (2) $\qquad E_x = 3689\text{ N} \qquad E_y = 1718.3\text{ N}$

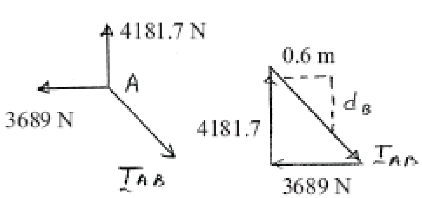
(*a*) $\qquad$ **E** $= 4069.6\text{ N} \angle 24.98°$ ◀

(*b*) cable: $\qquad \rightarrow \Sigma F_x = 0: -A_x + E_x = 0 \qquad A_x = E_x = 3689\text{ N}$

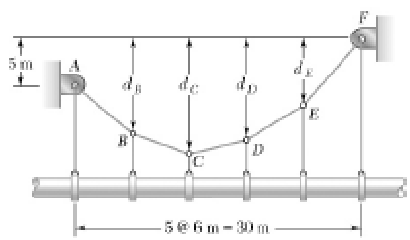
$$\uparrow \Sigma F_y = 0: A_y - 1600\text{ N} - 3200\text{ N} - 1100\text{ N} + 1718.3\text{ N} = 0$$

$$A_y = 4181.7\text{ N}$$

Point A:



$$\frac{d_B}{0.6\text{ m}} = \frac{4181.7\text{ N}}{3689\text{ N}} \qquad d_B = 0.680\text{ m}$$ ◀



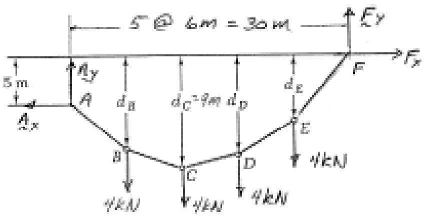
PROBLEM 7.95

Solve Prob. 7.94 assuming that $d_C = 9$ m.

SOLUTION

Note: 4 kN loads at A and F act directly on supports, not on cable.

FBD Cable:



$$\circlearrowleft \Sigma M_A = 0: (30\text{ m})F_y - (5\text{ m})F_x$$

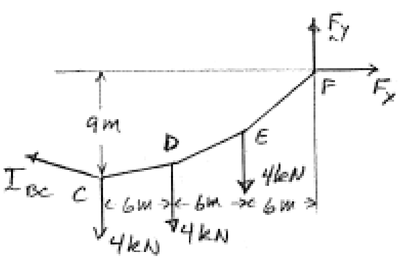
$$-(6\text{ m})\left[1(4\text{ kN}) + 2(4\text{ kN}) + 3(4\text{ kN}) + 4(4\text{ kN})\right] = 0$$

$$F_x - 6F_y = -48\text{ kN} \qquad (1)$$

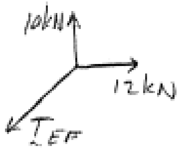
$$\circlearrowleft \Sigma M_C = 0: (18)F_y - (9\text{ m})F_x - (12\text{ m})(4\text{ kN}) - (6\text{ m})(4\text{ kN}) = 0$$

$$F_x - 2F_y = -8\text{ kN} \qquad (2)$$

FBD CDEF:



Solving (1) and (2) $\qquad \mathbf{F}_x = 12\text{ kN} \longrightarrow \qquad \mathbf{F}_y = 10\text{ kN} \uparrow$



$$T_{EF} = \sqrt{(10\text{ kN})^2 + (12\text{ kN})^2} = 15.62\text{ kN}$$

Since slope $EF >$ slope AB this is T_{max}

(*a*) $\qquad T_{max} = 15.62\text{ kN}$ ◀

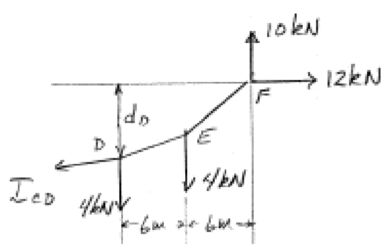
Also could note from FBD cable

$$\uparrow \Sigma F_y = 0: A_y + F_y - 4(4\text{ kN}) = 0$$

$$A_y = 16\text{ kN} - 12\text{ kN} = 4\text{ kN}$$

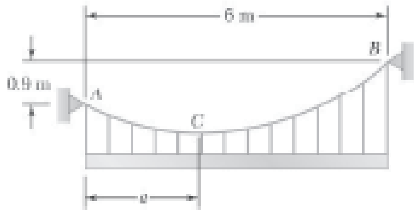
Thus $\qquad A_y < F_y \qquad$ and $\qquad T_{AB} < T_{EF}$

FBD DEF:



(*b*) $\qquad \circlearrowleft \Sigma M_D = 0: (12\text{ m})(10\text{ kN}) - d_D(12\text{ kN}) - (6\text{ m})(4\text{ kN}) = 0$

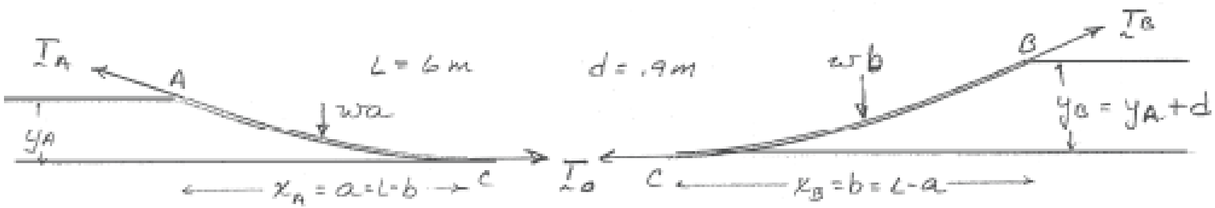
$$d_D = 8.00\text{ m} \blacktriangleleft$$



PROBLEM 7.112

Chain AB supports a horizontal, uniform steel beam having a mass per unit length of 85 kg/m. If the maximum tension in the chain is not to exceed 8 kN, determine (*a*) the horizontal distance a from A to the lowest point C of the chain, (*b*) the approximate length of the chain.

SOLUTION



$\circlearrowleft \Sigma M_A = 0: \quad y_A T_0 - \dfrac{a}{2} wa = 0$ $\qquad\qquad$ $\circlearrowleft \Sigma M_B = 0: \quad -y_B T_0 + \dfrac{b}{2} wb = 0$

$$y_A = \frac{wa^2}{2T_0} \qquad\qquad y_B = \frac{wb^2}{2T_0}$$

$$d = (y_B - y_B) = \frac{w}{2T_0}\left(b^2 - a^2\right)$$

But $\qquad T_0 = \sqrt{T_B^2 - (wb)^2} = \sqrt{T_{\max}^2 - (wb)^2}$

$$\therefore \quad (2d)^2\left[T_{\max}^2 - (wb)^2\right] = w^2\left(b^2 - a^2\right)^2 = L^2w^2\left(4b^2 - 4Lb + L^2\right)$$

or

$$4\left(L^2 + d^2\right)b^2 - 4L^3b + \left(L^4 - 4d^2\frac{T_{\max}^2}{w^2}\right) = 0$$

Using $\qquad L = 6$ m, $\quad d = 0.9$ m, $\quad T_{\max} = 8$ kN, $\quad w = (85\text{ kg/m})(9.81\text{ m/s}^2) = 833.85$ N/m

yields $\qquad b = (2.934 \pm 1.353)$ m, $\quad b = 4.287$ m $\quad (\text{since } b > 3\text{ m})$

(*a*) $\qquad a = 6\text{ m} - b = 1.713\text{ m}$ ◀