**CONCORDIA UNIVERSITY**
**Department of Mathematics & Statistics**

| **Course** | **Number** | **Section(s)** |
|---|---|---|
| Mathematics | 203 | All |
| **Examination** | **Date** | **Pages** |
| Final | December 2007 | 3 |
| **Instructors** | | **Course Examiner** |
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**Special Instructions**

▷ **Only approved calculators are allowed.**

*MARKS*

[9] **1.** (a) Sketch the graph of the function $f(x) = |(x-1)^2 - 1|$ starting from the graph of the standard parabola and using appropriate transformations.

(b) Suppose $f(x) = \dfrac{2x+1}{x+1}$ and $g(x) = \dfrac{x-1}{2-x}$. Find $f \circ g$ and $g \circ f$.

(c) Solve for $x$:

$$3^{\log_3(x^2)} = 2e^{\ln x} + 4 \cdot 10^{\log_{10}(2)}$$

[12] **2.** Evaluate the limits:

(a) $\displaystyle\lim_{t\to -1} \frac{t^2+3t+2}{t^2-t-2}$ (b) $\displaystyle\lim_{x\to 9} \frac{9x-x^2}{3-\sqrt{x}}$ (c) $\displaystyle\lim_{x\to -\infty} \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Do not use l'Hopital's rule.

[10] **3.** (a) Consider the function $f(x) = \dfrac{x^2-x-6}{|x-3|}$.

Calculate both one-sided limits at the point(s) where the function is undefined.

**3.** (b) Find parameters $a$ and $b$ such that the function

$$f(x) = \begin{cases} \cos x & \text{if} \quad x \leq 0 \\ ax + b & \text{if} \quad 0 < x \leq 3 \\ x^2 - 2 & \text{if} \quad x > 3 \end{cases}$$

will be continuous at every point. Sketch the graph of this function.

[15] **4.** Find derivatives of the functions (do not simplify the answer):

(a) $f(x) = (x + x^{-1})^2 \cos 2x$

(b) $f(x) = \dfrac{\sin^{-1}(\sqrt{1-x^2})}{\sqrt{1-x^2}}$

(c) $f(x) = (x^2)^\pi + \pi^{x^2}$

(d) $f(x) = 4x\sqrt{x + \sqrt{x}}$

(e) $f(x) = (\tan^{-1}(2x))^{\ln x}$ (use logarithmic differentiation).

[12] **5.** (a) If $f(x) = (1 + x)^n$, find the linearization $L(x)$ of $f(x)$ at $a = 0$ and use $L(x)$ to estimate $(1.003)^{50}$.

(b) Answer part (a) using differentials, that is, identify $dx$ and calculate $df$.

(c) If $g(x) = (x - 1)^2$, use the definition of the derivative to find $g'(3)$.

(d) Use the appropriate differentiation rule(s) to verify your answer to part (c) .

[18] **6.** (a) The equation of a curve defined implicitly is $x - 2y^2 + 5 = 3e^{x/y}$. Verify that the point $(0, 1)$ belongs to the curve. Find an equation of the tangent line to the curve at this point.

(b) Let $f(x) = x^2 + 2x - 1$ . Find a number $c$ that satisfies the Mean Value Theorem for the function $f(x)$ on $[0, 1]$.

(c) Use l'Hopital's rule to evaluate $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x^3}$ .

[10] **7.** (a) A plane is located at $x = 40$ km (horizontally) away from an airport at an altitude of $h$ km. At time $t = t_0$ radar at the airport detects the distance $s(t)$ between the plane and the airport decreasing at the rate $s'(t_0) = -400$ km/h (the plane is flying towards the airport). If the plane is maintaining a constant altitude of $h = 4$ km what is the speed $x'(t_0)$ of the aircraft at time $t_0$?

(b) A rectangular plot of land is to be bounded on one side by a river and on the other three sides by a fence. If there are 800 m of fencing available, what is the largest area that can be enclosed, and what are its dimensions?

[14] **8.** Given the function $f(x) = 4x^3 - x^4$,

(a) Find the domain and check for symmetry. Find asymptotes (if any).

(b) Calculate $f'(x)$ and use it to determine interval(s) where the function is increasing, interval(s) where the function is decreasing, and local extrema (if any).

(c) Calculate $f''(x)$ and use it to determine interval(s) where the function is concave upward, interval(s) where the function is concave downward and inflection point(s) (if any).

(d) Sketch the graph of the function.

[5] **Bonus Question**

Let

$$f(x) = \begin{cases} -x^3 & \text{if } x \geq 0 \\ x^3 & \text{if } x < 0 \end{cases}$$

Use the definition of the derivative to show that $f$ is differentiable at $x = 0$ and find $f'(0)$.