Assignment # 1

Q1

$y = 6x + 3$

$y = -11 + \frac{1}{6}x$ are parallel, prependicular, or other?

$y = 6x + 3 \Rightarrow m_1 = 6$

$y = -11 + \frac{1}{6}x \Rightarrow m_2 = \frac{1}{6}$

Then, the two lines are ~~not~~ neither parallel nor prependicular.

Q2

find the center and the radius of the circle

$x^2 + 7x + y^2 - 10y + 12 = 0$

we have to modify the equation to be in its standard form

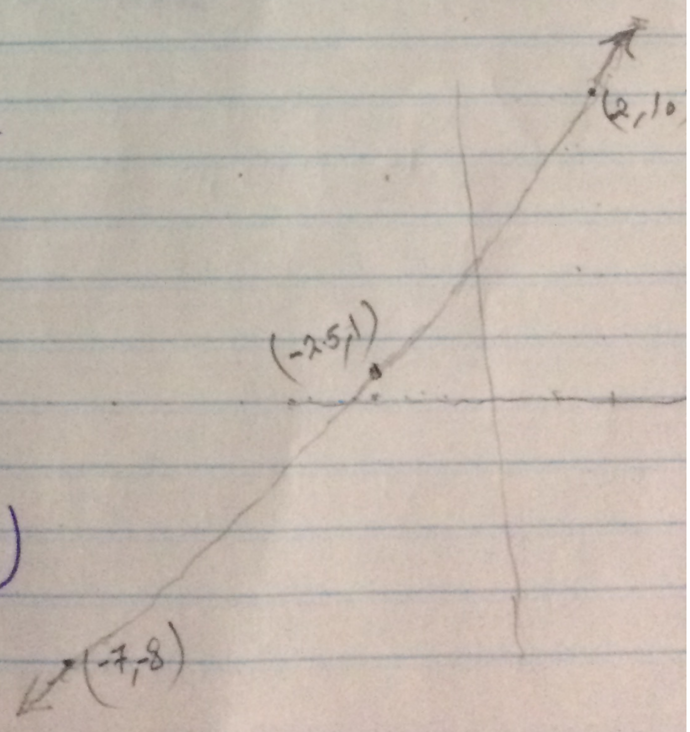
$$\left(x + \frac{7}{2}\right)^2 - \frac{49}{4} + (y - 5)^2 - 25 + 12 = 0$$

$$\left(x + \frac{7}{2}\right)^2 + (y - 5)^2 = \frac{101}{4}$$

Center: $(-3.5, 5)$

Radius: 5.025

Q3 find the distance between points (2,10) and (-7,-8) and the mid-point between them.



$$d = \sqrt{(2-(-7))^2 + (10-(-8))^2}$$

$$= \sqrt{81 + 324}$$

$$= \sqrt{405}$$

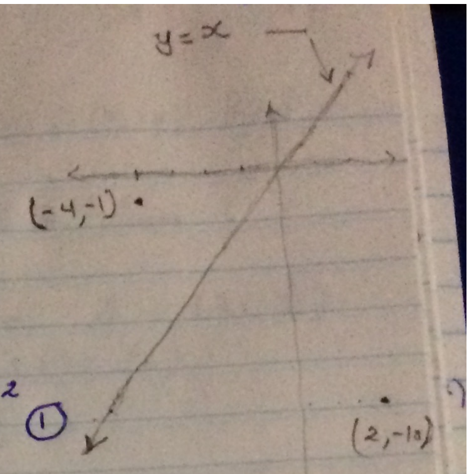
$$= 20.125$$

$$\text{Mid-Point} = \left(\frac{x_1 + x_2}{2}, \frac{y_2 - y_1}{2}\right)$$

$$= \left(-5/2, 1\right)$$

Q_4

Find the point (x,y) on line $y=x$ that is ~~lat~~ equidistant from the point $(-4,-1)$, $(2,-10)$



* the distance between (x,y) and $(-4,-1)$ Can be Calculated by

$$d_1 = \sqrt{(x+4)^2 + (y+1)^2} \Rightarrow (x+4)^2 + (y+1)^2 = d_1^2 \quad ①$$

* the distance between (x,y) and $(2,-10)$ Can be Calculated by

$$d_2 = \sqrt{(x-2)^2 + (y+10)^2} \Rightarrow (x-2)^2 + (y+10)^2 = d_2^2$$

Since $d_1 = d_2 \Rightarrow d_1^2 = d_2^2$ then

$$(x+4)^2 + (y+1)^2 = (x-2)^2 + (y+10)^2$$

Since (x,y) lies on the line $y=x$, then

$$(x+4)^2 + (x+1)^2 = (x-2)^2 + (x+10)^2$$

$$\Rightarrow \cancel{x^2} + 8x + 16 + \cancel{x^2} + 2x + 1 = \cancel{x^2} - 4x + 4 + \cancel{x^2} + 20x + 100$$

$$\Rightarrow 10x + 17 - 16x - 104 = 0$$

$$\Rightarrow -6x - 87 = 0 \quad \Rightarrow x = -14.5 \text{, then}$$

The point is $(-14.5, -14.5)$

Q5 the midpoint of AB is at (2, 2). If $A = (-10, 10)$, find B.

Since (2, 2) is the midpoint of AB, then

$$(2,2) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$ where A is (x_1, y_1), B is (x_2, y_2)

$$(2,2) = \left(\frac{-10+x_2}{2}, \frac{10+y_2}{2}\right)$$

$\Rightarrow$ $\frac{-10+x_2}{2} = 2 \Rightarrow x_2 - 10 = 4 \Rightarrow x_2 = 14$

$\frac{10+y_2}{2} = 2 \Rightarrow y_2 + 10 = 4 \Rightarrow y_2 = -6$

$\Rightarrow$ B is $(14, -6)$

Q7 find the center and the radius of a circle given by the equation

$$x^2+y^2+2x+10y+17=0$$

$$(x+1)^2 - 1 + (y+5)^2 - 25 + 17 = 0$$

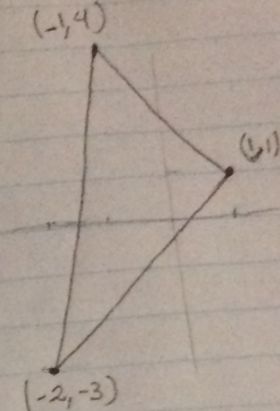
$$(x+1)^2 + (y+5)^2 = 9$$

Then,

the center is $(-1, -5)$

the radius $= 3$

Q6 Find the perimeter of the triangle with vertices at $(1,1)$, $(-1,4)$, and $(-2,-3)$



Answer

The perimeter of the triangle is the sum of the distances around it

Let d_1 is the distance between $(-1,4)$ and $(1,1)$ then

$$d_1 = \sqrt{(-1-1)^2+(4-1)^2}$$
$$= \sqrt{4+9}$$
$$= \sqrt{13}$$
$$= 3.61$$

Let d_2 is the distance between $(1,1)$ and $(-2,-3)$ then

$$d_2 = \sqrt{(1+2)^2+(1+3)^2}$$
$$= \sqrt{9+16}$$
$$= \sqrt{25}$$
$$= 5$$

Let d_3 is the distance between $(-1,4)$ and $(-2,-3)$

$$d_3 = \sqrt{(-1+2)^2+(4+3)^2}$$
$$= \sqrt{1+49}$$
$$= \sqrt{50}$$
$$= 7.07$$

Then, the perimeter of the triangle $= 3.61 + 5.83 + 7.07$
$= 15.68$

Q8 find an equation of the line passing through the point (6, 3) and prependicular to the line $y = 2x - 10$

Sol.

Since we can determine the equation of the line by using a point and its slope, so using the property that the line ~~and~~ $y = 2x - 10$ and our line are prependicular, then

$m_1 * m_2 = -1$, where

m_1 is the slope of our line

m_2 is the slope of the line $y = 2x - 10$

$m_2 = 2$

$\Rightarrow m_1 = -\frac{1}{2}$, then

- $y - y_1 = m_1 (x - x_1)$

$y - 3 = -\frac{1}{2}(x - 6)$

$\Rightarrow 2y - 6 = -x + 6$

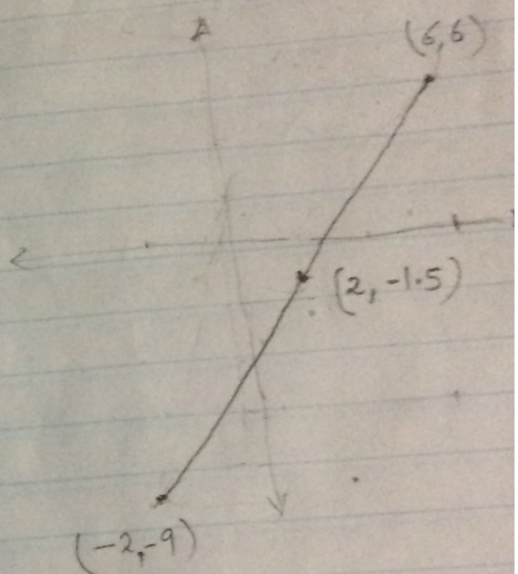
$\Rightarrow \boxed{2y + x - 12 = 0}$

$y = 6 - \frac{x}{2}$

Q9 Find the standard form for the equation of a circle $(x-h)^2+(y-k)^2=r^2$ with a diameter that has endpoints $(-2,-9)$ and $(6,6)$

Sol.

Since $(-2,-9)$ and $(6,6)$ is the endpoints of the diameter, then the midpoint in between is the center (h,k) and the distance between them is $2r$



$$\text{Midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$= \left(\frac{4}{2}, \frac{-3}{2}\right)$$

$$= (2, -1.5)$$

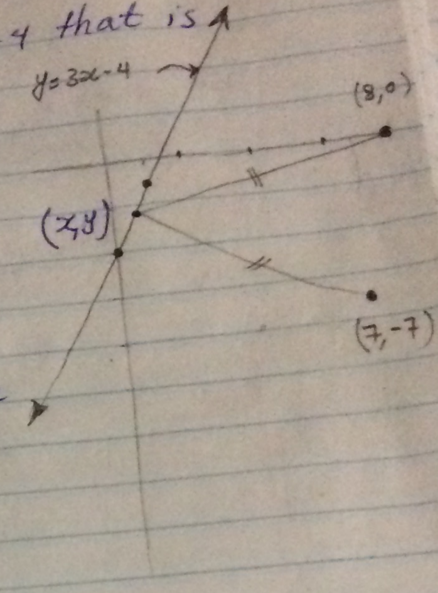
$$r = \sqrt{(-2-2)^2+(9+1.5)^2}$$

$$= \sqrt{\cancel{64+225}} = \sqrt{16+56.25}$$

$$r = \cancel{\frac{\sqrt{73}}{2}} \; \cancel{\frac{\sqrt{289}}{2}} = \sqrt{72.25}$$

$$= \cancel{4.27} = 8.5$$

Q10 Find the point (x,y) on the line $y=3x-4$ that is equistant from the point $(7,-7)$ and $(8,0)$



Answer

The distance between (x,y) and $(8,0)$ Can be Calculated as follows:

$$d_1 = \sqrt{(x-8)^2 + y^2} \longleftarrow ①$$

the distance between (x,y) and $(7,-7)$ Can be Calculated as follows

$$d_2 = \sqrt{(x-7)^2 + (y+7)^2} \longleftarrow ②$$

from ① and ②

$$d_1 = d_2 \Rightarrow d_1^2 = d_2^2$$

then,

$$(x-8)^2 + y^2 = (x-7)^2 + (y+7)^2$$

Since (x,y) lies on the line $y=3x-4$, so we Can subsititute every y by $3x-4$

$$(x-8)^2 + (3x-4)^2 = (x-7)^2 + (3x-4+7)^2$$

$$(x-8)^2 + (3x-4)^2 = (x-7)^2 + (3x+3)^2$$

$$\cancel{x^2} - 16x + 64 + \cancel{9x^2} - 24x + 16 = \cancel{x^2} - 14x + 49 + \cancel{9x^2} + 18x + 9$$

$$80 - 40x = 58 + 4x$$

$$44x = 22 \Rightarrow \boxed{x = 0.5}$$

Since $y = 3x-4$, then

$$\boxed{y = -2.5}$$