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Total Marks: 70 marks

1. For each of the following questions, write the correct answer in the space provided. No justifications are necessary.

(a) [2 marks] $\lim_{x \rightarrow \infty} \frac{2}{(x-1)^2} = \underline{\hspace{2cm}}$

(b) [2 marks] $\lim_{x \rightarrow 3^+} \ln(x^2 - 9) = \underline{\hspace{2cm}}$

(c) [2 marks] $\lim_{x \rightarrow 0} \frac{\tan(2x)}{\sin x} = \underline{\hspace{2cm}}$

(d) [2 marks] If $f(x) = \begin{cases} x & \text{if } x < 0 \\ e^x & \text{if } x \geq 0 \end{cases}$, then $f(x)$ is not continuous at $x = \underline{\hspace{2cm}}$

2. [5 marks] Suppose f is a function such that $\frac{\ln(1+x)}{x} \leq f(x) \leq \frac{x^2 + \ln(1+x)}{x}$ for all $x \in (0, \infty)$.

Use the Squeeze Theorem to evaluate $\lim_{x \rightarrow 0^+} f(x)$.

3. (a) [6 marks] Evaluate $\lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$.

- (b) [6 marks] Find all the horizontal and vertical asymptotes of the graph of the function

$$y = \frac{x^2 - x}{x^2 - 6x + 5}.$$

4. [8 marks] Use the Intermediate Value Theorem to show that the equation $\cos x = x^3$ has at least one real solution.

5. [8 marks] Find an equation of the tangent line to the curve defined implicitly by the equation

$$2(x^2 + y^2)^2 = 25(x^2 - y^2) \text{ at the point } (3, 1).$$

6. [19 marks] Find $\frac{dy}{dx}$ for the following functions.

(a) $y = \cos\left(\frac{1 - e^{2x}}{1 + e^{2x}}\right)$

(b) $y = x^{2x}$, $x > 0$

(c) $y = \sqrt{1 - x^2} \cdot \arccos(x)$

(d) $y = \log_5(x + \sqrt{x^2 - 1})$

7. [5 marks] Find the linear approximation of the function $y = \sqrt{1-x}$ at $x = 0$ and use it to approximate $\sqrt{0.99}$ and $\sqrt{1.01}$.

8. [5 marks] Find the absolute maximum and absolute minimum values of the function:

$$y = 2 \cos t + \sin 2t, \quad \left[0, \frac{\pi}{2}\right].$$