Total Marks: 70 marks

1. For each of the following questions, write the correct answer in the space provided. No justifications are necessary.

   (a) [2 *marks*] $\lim\limits_{x\to\infty} \dfrac{2}{(x-1)^2} =$ _____

   (b) [2 *marks*] $\lim\limits_{x\to 3^+} \ln\left(x^2-9\right) =$ _____

   (c) [2 *marks*] $\lim\limits_{x\to 0} \dfrac{\tan(2x)}{\sin x} =$ _____

   (d) [2 *marks*] If $f(x) = \begin{cases} x & \text{if } x<0 \\ e^x & \text{if } x\geq 0 \end{cases}$, then $f(x)$ is not continuous at $x =$ _____

2. [5 *marks*] Suppose f is a function such that $\dfrac{\ln(1+x)}{x} \leq f(x) \leq \dfrac{x^2+\ln(1+x)}{x}$ for all $x \in (0,\infty)$. Use the Squeeze Theorem to evaluate $\lim\limits_{x\to 0^+} f(x)$.

3. (a) [6 *marks*] Evaluate $\lim\limits_{t\to 0}\left(\dfrac{1}{t\sqrt{1+t}} - \dfrac{1}{t}\right)$.

   (b) [6 *marks*] Find all the horizontal and vertical asymptotes of the graph of the function
   $$y = \frac{x^2-x}{x^2-6x+5}.$$

4. [8 *marks*] Use the Intermediate Value Theorem to show that the equation $\cos x = x^3$ has at least one real solution.

5. [8 *marks*] Find an equation of the tangent line to the curve defined implicitly by the equation
   $$2\left(x^2+y^2\right)^2 = 25\left(x^2-y^2\right) \text{ at the point } (3,1).$$

6. [19 *marks*] Find $\dfrac{dy}{dx}$ for the following functions.

   (a) $y = \cos\left(\dfrac{1-e^{2x}}{1+e^{2x}}\right)$

   (b) $y = x^{2x},\ x>0$

   (c) $y = \sqrt{1-x^2}\cdot\arccos(x)$

   (d) $y = \log_5\left(x+\sqrt{x^2-1}\right)$

7. [5 *marks*] Find the linear approximation of the function $y = \sqrt{1-x}$ at $x=0$ and use it to approximate $\sqrt{0.99}$ and $\sqrt{1.01}$.

8. [5 *marks*] Find the absolute maximum and absolute minimum values of the function:
   $$y = 2\cos t + \sin 2t, \quad \left[0, \frac{\pi}{2}\right].$$