1L03/1L03E Fall 2016

Test #1 Full Solutions

Versions 1-4

Test #1 Solutions – Version #1

Multiple Choice Key:

Question #	1	2	3	4	5	6	7	8
Answer Letter	B	A	D	B	B	B	A	C

Multiple Choice – Worked Solution:

1. $(S \cap T)' \cup T$ $= S' \cup T' \cup T$ – Apply DeMorgan's Law

$= S' \cup U$ – Since $T' \cup T = U$

$= U$ **Ans: B**

2. **(**5 burgers)*(4 sides)*(3 drinks) = 60 meals **Ans: A**

3. All elements of either S or T that are in R: $(S \cup T) \cap R$ **Ans: D**

4. $n(R \cup T) = n(R) + n(T) - n(R \cap T)$

$20 = 10 + n(T) - 5$

$n(T) = 15$ **Ans: B**

5. Given n stands, we have P(10,n) displays, so P(10,n) = 5,040 = P(10,4) **Ans: B**

6. U = {1,2,3,5,7,8,9}, T = {1,3,5,8}

so T' = {all elements of U not in T} = {2,7,9} **Ans: B**

7. $n(E) = n(F \cup E)$ So every element of F is also an element of E (ie union adds no new elements)

So F is a subset of E, that is: $F \subseteq E$ **Ans: A**

8. $T = \{5,7,8\}$, and $E \subseteq T$. So E only contains elements of T, like {5,7} **Ans: C**

"Long" Answer Question Solutions

9. **a)** 5 letters, 30 possibilities for each letter = 30*30*30*30*30 = 30^5 = 24,300,000 plates.

b) The middle letter can be any letter (30 possibilities)

If the first two letters are the same, 30 possibilities for the first one, one for the second. And 29 for each of the last two.

If the first two letters are different, 30*29 possibilities for the first two, 28 for the last two
So: (30*1)*30*29*29 + (30*29)*30*28*28 = 21,219,300

c) Select 5 letters from 30, in an ordered manner: P(30,5) = 30*29*28*27*26 = 17,100,720

d) Two plates are distinct if and only if they have different letters.
So just choose the letters:
Select 5 letters from 30, without considering order: C(30,5) = 30*29*28*27*26/5! = 142,506

Marking: 1 mark for each a perfect answer.
1/2 mark if close (eg, in b only having the one case, or in c,d mixing up P and C)
0 marks if final number only, without any calculation.

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10. **a)** Elements of $A \cap B$ are cars that are both new and expensive.
Elements of $(A \cap B) \cup C$ are either new, expensive cars or are red cars

b) Inexpensive = not expensive, ie. cars in B'
New cars are in set A
Cars that are not red are in C'
Inexpensive, new cars which are not red are in the intersection of all three, so we get:
$B' \cap A \cap C'$

Marking: 1 mark for each a perfect answer.
1/2 mark if close (eg,at most one error)

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11. **a)** S'' = S = {white, black, blue}
b) $S \cap T'$ means in S and not in T, so $S \cap T'$ = {white}
c) {red, blue}, {red}, {blue}, { } or $\varnothing$.

Marking: 1/2 mark for a)
1/2 mark for b)
1/2 mark for each set in c) (subtract one half mark for each extra "set" included)

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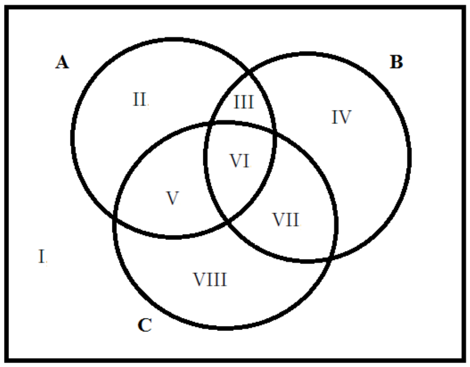
12. a) First, for ease of reference, let's draw our sets as a Venn diagram, with the basic regions indicated:



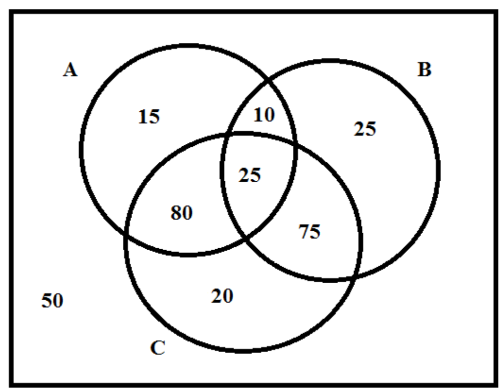
Each of sets (circles) A, B and C represent customers that buy fertilizer A, B and C respectively.

Let's now convert our statements about students into statements about the number of students in each basic region. Note that for brevity, we'll use the Roman numeral to represent the number of elements in that region.

300 total customers:	I + II + III + IV + V + VI + VII + VIII = 300
25 buy all three brands:	VI = 25
130 customers buy brand A:	II + III + V + VI = 130
135 customers buy brand B:	III + IV + VI + VII = 135
100 don't buy brand C:	I + II + III + IV = 100
50 customers don't buy any fertilizer :	I = 50
35 customers buy both B and A :	III + VI = 35
105 customers buy both A and C:	V + VI = 105

We're given **I =50** and **VI = 25**
Since VI = 25, the last two equations give us: **III = 10, V = 80**
From brand A: II + 10 + 80 + 25 = 130 so **II = 15**
From not brand C: 50 + 15 + 10 +IV = 100 so **IV = 25**
From brand B: 10 + 25 + 25 + VII = 135 so **VII = 75**
From total: 50+15+10+25+80+25+75+VIII=300
so **VIII = 20**

So our diagram becomes the one pictured to the right:



b) Exactly two kinds of fertilizer means regions III+V+VII = 10 + 80+ 75 = 165 customers.

Marking: a) 2 marks total:
1/2 mark for an appropriately drawn Venn diagram (Any orientation)
3/2 mark for valid calculations and final values.
(Note, a valid Venn diagram, with correct values is worth at most 1 mark if there are no calculations or justification.)
b) 1 mark total: 1/2 mark for correct regions, 1/2 mark for correct value.
(Note value without indication of where it came from is worth 1/2 mark only)

Test #1 Solutions – Version #2

Multiple Choice Key:

Question #	1	2	3	4	5	6	7	8
Answer Letter	**B**	**C**	**A**	**C**	**B**	**A**	**C**	**D**

Multiple Choice – Worked Solution:

1. U = {1,2,3,5,7,8,9}, T = {2,3,7,8}
so T' = {all elements of U not in T} = {1,5,9} **Ans: B**

2. n(R ∪ T) = n(R) + n(T) – n(R ∩ T)
20 = 15 + n(T) – 5
n(T) = 10 **Ans: C**

3. (S ∪ T)' ∩ T = S' ∩ T' ∩ T – Apply DeMorgan's Law
= S' ∩ ∅ – Since T' ∩ T = ∅
= ∅ **Ans: A**

4. **(**5 skirts)*(3 shirts)*(4 shoes) = 60 outfits **Ans: C**

5. Given n spots, we have P(10,n) ordered lists, so P(10,n) = 30,240= P(10,5) **Ans: B**

6. n(F) = n(F ∪ E) So every element of E is also an element of F (ie union adds no new elements)
So E is a subset of F, that is: E ⊆ F **Ans: A**

7. $T = \{5,7,8\}$, and $E \subseteq T$. So E only contains elements of T, like {5,7} **Ans: C**

8. All elements of either S or R that are in T: (S ∪ R) ∩ T **Ans: D**

“Long” Answer Question Solutions

9. a) 5 letters, 30 possibilities for each letter = 30*30*30*30*30 = 30^5 = 24,300,000 plates.

b) The middle letter can be any letter (30 possibilities)

If the first two letters are the same, 30 possibilities for the first one, one for the second. And 29 for each of the last two.

If the first two letters are different, 30*29 possibilities for the first two, 28 for the last two
So: (30*1)*30*29*29 + (30*29)*30*28*28 = 21,219,300

c) Select 5 letters from 30, in an ordered manner: P(30,5) = 30*29*28*27*26 = 17,100,720

d) Two plates are distinct if and only if they have different letters.
So just choose the letters:
Select 5 letters from 30, without considering order: C(30,5) = 30*29*28*27*26/5! = 142,506

Marking: 1 mark for each a perfect answer.
1/2 mark if close (eg, in b only having the one case, or in c,d mixing up P and C)
0 marks if final number only, without any calculation.

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10. a) S′′ = S = {b, c, d}
b) S’ ∩ T means not in S and in T, so S ∩ T’ = {e}
c) {c, e}, {c}, {e}, { } or ∅.

Marking: 1/2 mark for a)
1/2 mark for b)
1/2 mark for each set in c) (subtract one half mark for each extra “set” included)

---

11. a) Elements of A ∩ B are shirts that are both purple and expensive.
Elements of (A ∩ B) ⋃ C are either purple, expensive shirts or are fashionable shirts.

b) Inexpensive = not expensive, ie. shirts in B’
Fashionable shirts are in set C
Shirts that are not purple are in A’
Inexpensive, fashionable shirts which are not purple are in the intersection of all three, so we get:
B’ ∩ C ∩ A’

Marking: 1 mark for each a perfect answer.
1/2 mark if close (eg, at most one error)

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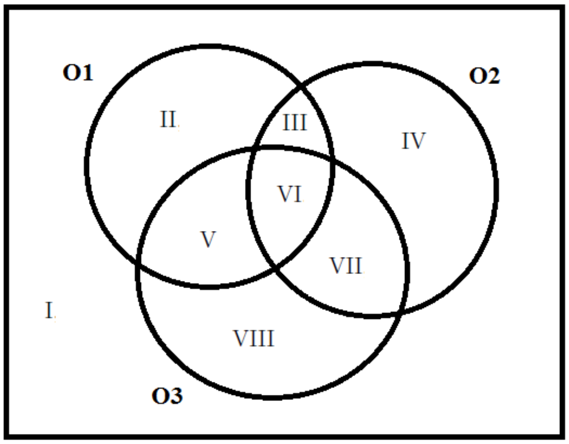
12. a) First, for ease of reference, let's draw our sets as a Venn diagram, with the basic regions indicated:



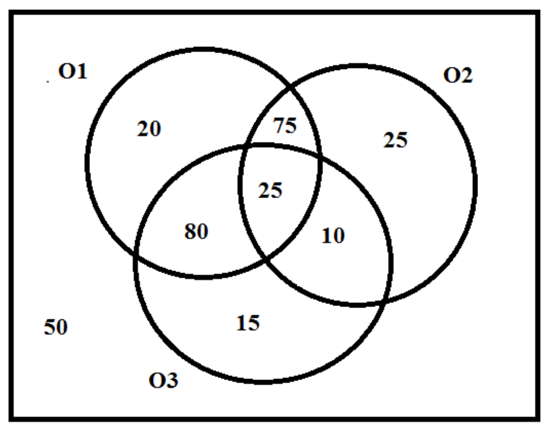
Each of sets (circles) O1, O2 and O3 represent players that unlock objective 1, 2 and 3, respectively.

Let's now convert our statements about students into statements about the number of students in each basic region. Note that for brevity, we'll use the Roman numeral to represent the number of elements in that region.

300 total players:	I + II + III + IV + V + VI + VII + VIII = 300
50 fail to unlock any objectives:	I = 50
170 fail to unlock objective #3:	I + II + III + IV = 170
200 unlock objective #1:	II + III + V + VI = 200
135 unlock objective #2:	III + IV + VI + VII = 135
100 unlock both #1 and #2:	III + VI = 100
105 unlock both #1 and #3:	V + VI = 105
25 unlock all three objectives:	VI = 25

We're given **I =50** and **VI = 25**
Since VI = 25, the next to the last two equations give us:
III = 75, V = 80
From objective O1: II + 75 + 80 + 25 = 200 so **II = 20**
From failing O3: 50 + 20 + 75 + IV = 170 so **IV = 25**
From objective O2 75 + 25 + 25 + VII = 135 so **VII = 10**
From total: 50+20+75+25+80+25+10+VIII=300
so **VIII = 15**

So our diagram becomes the one pictured to the right:



b) Failing to unlock exactly one objective means unlocking exactly two:
III + V + VII = 75+ 80+ 10 = 165 players.

Marking: a) 2 marks total:
1/2 mark for an appropriately drawn Venn diagram (Any orientation)
3/2 mark for valid calculations and final values.
(Note, a valid Venn diagram, with correct values is worth at most 1 mark if there are no calculations or justification.)
b) 1 mark total: 1/2 mark for correct regions, 1/2 mark for correct value.
(Note value without indication of where it came from is worth 1/2 mark only)

Test #1 Solutions – Version #3

Multiple Choice Key:

Question #	1	2	3	4	5	6	7	8
Answer Letter	E	C	A	A	A	A	C	A

Multiple Choice – Worked Solution:

1. Given n stands, we have P(10,n) displays, so P(10,n) = 5,040 = P(10,4) **Ans: E**

2. All elements of either S or T that are in R: $(S \cup T) \cap R$ **Ans: C**

3. $n(R \cup T) = n(R) + n(T) - n(R \cap T)$

$20 = 15 + n(T) - 5$

$n(T) = 10$ **Ans: A**

4. $T = \{5,7,8\}$, and $E \subseteq T$. So E only contains elements of T, like {5,7} **Ans: A**

5. U = {1,2,3,5,7,8,9}, T = {1,3,5,8}

so T' = {all elements of U not in T} = {2,7,9} **Ans: A**

6. $n(F) = n(F \cup E)$ So every element of E is also an element of F (ie union adds no new elements)

So F is a subset of E, that is: $E \subseteq F$ **Ans: A**

7. $(S \cap T)' \cup T = S' \cup T' \cup T$ – Apply DeMorgan's Law

$= S' \cup U$ – Since $T' \cup T = U$

$= U$ **Ans: C**

8. **(**5 skirts)*(3 shirts)*(4 pairs of shoes) = 60 outfits **Ans: A**

“Long” Answer Question Solutions

9. a) 6 letters, 20 possibilities for each letter = 20*20*20*20*20 = 20^6 = 64,000,000 plates.

b) The middle two letters can be any letter (20*20 possibilities)

If the first two letters are the same, 20 possibilities for the first one, one for the second.
And 19 for each of the last two.

If the first two letters are different, 20*19 possibilities for the first two, 18 for the last two
So: (20*1)*20*20*19*19 + (20*19)*20*20*18*18 = 52,136,000

c) Select 6 letters from 20, in an ordered manner: P(20,6) = 20*19*18*17*16*15= 27,907,200

d) Two plates are distinct if and only if they have different letters.
So just choose the letters:
Select 6 letters from 20, without considering order: C(20,6) = 20*19*18*17*16*15/6! = 38,760

Marking: 1 mark for each a perfect answer.
1/2 mark if close (eg, in b only having the one case, or in c,d mixing up P and C)
0 marks if final number only, without any calculation.

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10. a) S′′ = S = {white, black, blue}
b) S’ ∩ T means not in S and in T, so S’ ∩ T = {red}
c) {red, blue}, {red}, {blue}, { } or ∅.

Marking: 1/2 mark for a)
1/2 mark for b)
1/2 mark for each set in c) (subtract one half mark for each extra “set” included)

---

11. a) Elements of A ∩ B are cars that are both new and expensive.
Elements of (A ∩ B) ∪ C are either new, expensive cars or are red cars

b) Inexpensive = not expensive, ie. cars in B’
New cars are in set A
Cars that are not red are in C’
Inexpensive, new cars which are not red are in the intersection of all three, so we get:
B’ ∩ A ∩ C’

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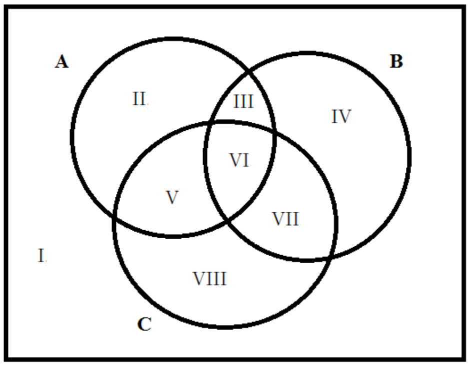
12. a) First, for ease of reference, let's draw our sets as a Venn diagram, with the basic regions indicated:



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Let's now convert our statements about students into statements about the number of students in each basic region. Note that for brevity, we'll use the Roman numeral to represent the number of elements in that region.

300 total customers:	I + II + III + IV + V + VI + VII + VIII = 300
25 buy all three brands:	VI = 25
130 customers buy brand A:	II + III + V + VI = 130
135 customers buy brand B:	III + IV + VI + VII = 135
100 don't buy brand C:	I + II + III + IV = 100
50 customers don't buy any fertilizer:	I = 50
35 customers buy both B and A:	III + VI = 35
105 customers buy both A and C:	V + VI = 105

We're given **I =50** and **VI = 25**

Since VI = 25, the last two equations give us: **III = 10, V = 80**

From brand A: II + 10 + 80 + 25 = 130 so **II = 15**

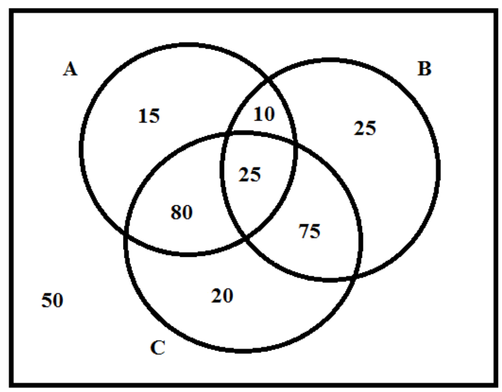
From not brand C: 50 + 15 + 10 +IV = 100 so **IV = 25**

From brand B: 10 + 25 + 25 + VII = 135 so **VII = 75**

From total: 50+15+10+25+80+25+75+VIII=300

so **VIII = 20**

So our diagram becomes the one pictured to the right:



b) Exactly two kinds of fertilizer means regions III+V+VII = 10 + 80+ 75 = 165 customers.

Marking: a) 2 marks total:

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(Note, a valid Venn diagram, with correct values is worth at most 1 mark if there are no calculations or justification.)

b) 1 mark total: 1/2 mark for correct regions, 1/2 mark for correct value.

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Test #1 Solutions – Version #4

Multiple Choice Key:

Question #	1	2	3	4	5	6	7	8
Answer Letter	E	B	E	D	C	C	C	B

Multiple Choice – Worked Solution:

1. All elements of either S or R that are in T: $(S \cup R) \cap T$ **Ans: E**

2. Given n spots, we have P(10,n) ordered lists, so P(10,n) = 30,240= P(10,5) **Ans: B**

3. U = {1,2,3,5,7,8,9}, T = {2,3,7,8}
so T' = {all elements of U not in T} = {1,5,9} **Ans: E**

4. $(S \cup T)' \cap T = S' \cap T' \cap T$ – Apply DeMorgan's Law

$= S' \cap \varnothing$ – Since $T' \cap T = \varnothing$

$= \varnothing$ **Ans: D**

5. (5 burgers)*(4 sides)*(3 drinks) = 60 meals **Ans: C**

6. $T = \{5,7,8\}$, and $E \subseteq T$. So E only contains elements of T, like {5,7} **Ans: C**

7. $n(E) = n(F \cup E)$ So every element of F is also an element of E (ie union adds no new elements)

So F is a subset of E, that is: $F \subseteq E$ **Ans: C**

8. $n(R \cup T) = n(R) + n(T) - n(R \cap T)$

20 = 10 + n(T) – 5
n(T) = 15 **Ans: B**

“Long” Answer Question Solutions

9. **a)** 5 letters, 30 possibilities for each letter = 30*30*30*30*30 = 30^5 = 24,300,000 plates.

b) The middle letter can be any letter (30 possibilities)

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10. **a)** S′′ = S = {b, c, d}
b) S’ ∩ T means in not in S and in T, so S’ ∩ T = {e}
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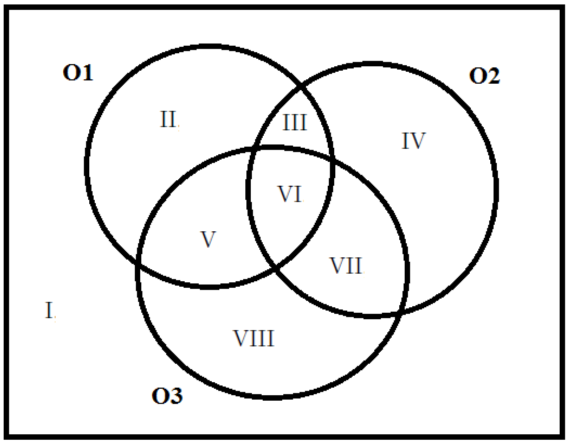

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300 total players:	I + II + III + IV + V + VI + VII + VIII = 300
50 fail to unlock any objectives:	I = 50
170 fail to unlock objective #3:	I + II + III + IV = 170
200 unlock objective #1:	II + III + V + VI = 200
135 unlock objective #2:	III + IV + VI + VII = 135
100 unlock both #1 and #2:	III + VI = 100
105 unlock both #1 and #3:	V + VI = 105
25 unlock all three objectives:	VI = 25

We're given **I =50** and **VI = 25**
Since VI = 25, the next to the last two equations give us:
III = 75, V = 80
From objective O1: II + 75 + 80 + 25 = 200 so **II = 20**
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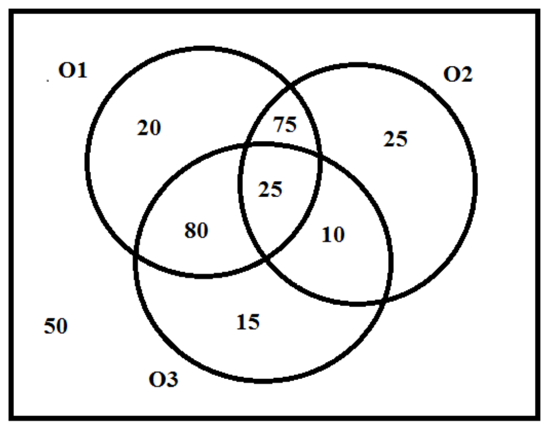
From objective O2 75 + 25 + 25 + VII = 135 so **VII = 10**
From total: 50+20+75+25+80+25+10+VIII=300
so **VIII = 15**

So our diagram becomes the one pictured to the right:



b) Failing to unlock exactly one objective means unlocking exactly two:
III + V + VII = 75+ 80+ 10 = 165 players.

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(Note, a valid Venn diagram, with correct values is worth at most 1 mark if there are no calculations or justification.)
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