

ENGR 233 MIDTERM OCT. 20, 2016

INSTRUCTIONS: Answer all six questions. The six questions are equally valued. Formula sheet is attached. Legal calculators are permitted.

1. Consider the 3-dimensional vector field $\vec{F} = \langle y^2, 2xy + e^{3z}, 3ye^{3z} \rangle$
- (a) Show that this field is conservative.
 - (b) Find a potential function.
 - (c) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the space-curve given by

$$\vec{r}(t) = \langle t^2 + t^3, e^{5t}, 7 \rangle, \quad 0 \leq t \leq 3$$

2. A mountain is described by the equation $z = f(x, y) = 7 - x^2 - 2y^2$ where z represents altitude, and a skier is standing at the point $(1, 1, 4)$.

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- (a) Find the directional derivative in the unit direction $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$.
 - (b) In which unit direction should the skier face if he or she wants to go downhill the fastest? What is the directional derivative in that direction?
 - (c) Sketch the level curve $f(x, y) = 4$ and draw the gradient of f at the point $(1, 1)$

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3. A cannon is on the edge of a 100 ft. cliff, and a cannonball is shot at a 45 degree upward angle with a speed of $\sqrt{18}$ ft/sec.

- (a) Find the time at which the cannonball hits the ground.
- (b) What is the horizontal range of the cannonball?

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4. Given the space-curve $\vec{r}(t) = \langle \cos 2t, \sin 2t, 6t \rangle$:

- (a) Find the arc-length as t goes from 0 to 1.
- (b) Find the curvature at $t = 0$.

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5. Consider the surface given by $z = 2x^3 - e^{xy}$.

- (a) Find the tangent plane at the point $(1, 0, 1)$.
- (b) Find the normal line at the point $(1, 0, 1)$.

6. (a) Find the area of the triangle with vertices $(1, 1, 1)$, $(3, 4, 5)$ and $(8, 1, 4)$.

- (b) Show that the following three vectors are coplanar:
 $\vec{a} = \langle 1, 4, -7 \rangle$, $\vec{b} = \langle 2, -1, 4 \rangle$, $\vec{c} = \langle 0, -9, 18 \rangle$.
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- $\wedge = 0$