

MAT 1322 C W2011 Wednesday, Feb. 2nd 17:30–19:00 Prof. C. Rada

## MIDTERM TEST 1

Max = 20

Student Number: \_\_\_\_\_

- Time: 80 min.
- Only basic scientific calculators are permitted (non-graphing, non-programmable, no integration or differentiation capabilities). Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. Do not use any other paper.
- The problems require complete and clearly presented solutions and carry part marks if there is substantial correct work towards the solution.
- There are five questions worth four marks each.

1. (a) Consider the integral  $\int_1^4 \frac{3}{(x-2)^{5/3}} dx$ . Does it converge or diverge? If it converges, give its value.

(b) Use the Comparison Test to determine if the integral  $\int_2^\infty \frac{3 + \sin x}{x^3 + 5x} dx$  converges or diverges.

**2.** Sketch the region bounded by the curves  $y = x^2$  and  $y = 8 - x^2$ . What is the area of the region?

**3.** Use the method of cylindrical shells to find the volume of the solid obtained when the region bounded by  $y = x$ ,  $y = 1/x$ ,  $x = 1$  and  $x = 3$  is rotated around the  $y$ -axis. Include a sketch of the region and a typical cylinder (with dimensions) in your solution.

4. A heavy rope of length 12 m has a density of 1.5 kg/m and is hanging over the edge of a tall building. How much work is done pulling the rope to the top of the building? The acceleration of gravity is  $g = 9.8 \text{ m/s}^2$ . Define clearly all the variables that enter into your solution and provide a drawing which shows their meaning.

5. (a) Give the integral for the average value of the function  $f(x) = \arcsin x$  on the interval  $[0, 1/2]$  and then evaluate it.

(b) Solve the initial value problem:  $(1 + x^2) \frac{dy}{dx} = 2y$ ,  $y(0) = 3$ .

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1. (a) Consider the integral  $\int_1^5 \frac{3}{(x-4)^{7/3}} dx$ . Does it converge or diverge? If it converges, give its value.

(b) Use the Comparison Test to determine if the integral  $\int_1^\infty \frac{4 - \cos x}{x^4 + 3x} dx$  converges or diverges.

**2.** Sketch the region bounded by the curves  $y = x^2$  and  $y = 18 - x^2$ . What is the area of the region?

**3.** Use the method of cylindrical shells to find the volume of the solid obtained when the region bounded by  $y = x$ ,  $y = 1/x$ ,  $x = 1$  and  $x = 2$  is rotated around the  $y$ -axis. Include a sketch of the region and a typical cylinder (with dimensions) in your solution.

4. A heavy rope of length 10 m has a density of 1.25 kg/m and is hanging over the edge of a tall building. How much work is done pulling the rope to the top of the building? The acceleration of gravity is  $g = 9.8 \text{ m/s}^2$ . Define clearly all the variables that enter into your solution and provide a drawing which shows their meaning.

5. (a) Give the integral for the average value of the function  $f(x) = \arctan x$  on the interval  $[0, \sqrt{3}]$  and then evaluate it.

(b) Solve the initial value problem:  $\sqrt{1-x^2} \frac{dy}{dx} = 3y$ ,  $y(0) = 4$ .