COMP1805 (Fall 2016) – "Discrete Structures I"

Quiz 2 of 4 – Practice Version

Student Number	*Student Name*

This is a closed book exam. No calculators, cellphones, laptops, or other aids are permitted. Answer all questions in the space provided. Show all your work because correct answers presented without justification may receive a mark of zero.

p $\therefore\ p \vee q$	}	**Addition**	$p \wedge q$ $\therefore\ p$	}	**Simplification**
p q $\therefore\ p \wedge q$	}	**Conjunction**	$p \vee q$ $\neg p \vee r$ $\therefore\ q \vee r$	}	**Resolution**
p $p \rightarrow q$ $\therefore\ q$	}	**Modus Ponens**	$\neg q$ $p \rightarrow q$ $\therefore\ \neg p$	}	**Modus Tollens**
$\neg p$ $p \vee q$ $\therefore\ q$	}	**Disjunctive Syllogism**	$p \rightarrow q$ $q \rightarrow r$ $\therefore\ p \rightarrow r$	}	**Hypothetical Syllogism**

1. Using **only the rules of inference** above (and none of the logical equivalences **except the implication equivalence**), show that the following argument is valid. Make sure that **you include both the inference rule and the line number(s)** to which that rule is applied, for each line of your argument. [6 marks]

(1) $a \rightarrow b$
(2) $(\neg b \vee c)$
(3) $\neg d \rightarrow \neg(\neg a \vee c)$

Show: $(e \vee d)$

COMP1805 (Fall 2016) – "Discrete Structures I"

Quiz 2 of 4 – Practice Version

Student Number									*Student Name*

This is a closed book exam. No calculators, cellphones, laptops, or other aids are permitted. Answer all questions in the space provided. Show all your work because correct answers presented without justification may receive a mark of zero.

$\forall x\, P(x)$ ∴ $P(c)$ } **Universal Instantiation**

$\exists x\, P(x)$ ∴ $P(c)$ } **Existential Instantiation for "fresh" variable c**

with "arbitrary" variable c

$P(c)$ ∴ $\forall x\, P(x)$ } **Universal Generalization**

$P(c)$ ∴ $\exists x\, P(x)$ } **Existential Generalization**

2. Using **only the rules of inference** (including the four above), show that the following argument is valid. You may assume that all the premises given are true. Make sure that you **include both the inference rule and the line number(s)** to which that rule is applied. You are also expected to **define each predicate**. Note that **each of the premises and the conclusion must be translated into quantified statements**. The **universe of discourse is all animals**. [9 marks]

Given: ***There is a polar bear that was born in the ocean.***
If an animal is born in the ocean then it can swim.
All polar bears eat seals.

Show: ***There is a polar bear that eats seals and can swim.***

Student Number	***Student Name***

This is a closed book exam. No calculators, cellphones, laptops, or other aids are permitted. Answer all questions in the space provided. Show all your work because correct answers presented without justification may receive a mark of zero.

3. Prove the following using either **a direct proof or an indirect proof**. Start your proof by **translating the claim into a quantified predicate logic expression** (assuming that the **universe of discourse is all integers**).

   Please also note that a number x is "even" if it can be expressed using $x = 2a$ for some integer a, and a number y is "odd" if it can be expressed using $y = 2b + 1$ for some integer b. [5 marks]

The product xy is odd if and only if x and y are both odd.

COMP1805 (Fall 2016) – "Discrete Structures I"

Quiz 2 of 4 – Practice Version

Student Number	***Student Name***

This is a closed book exam. No calculators, cellphones, laptops, or other aids are permitted. Answer all questions in the space provided. Show all your work because correct answers presented without justification may receive a mark of zero.

This page has been intentionally left blank and may be used if you require additional space.

If you do not require additional space you may contemplate the emptiness.