

Question (1)

The range of signed integers N expressed in 2's complement representation that can be stored in a 10-bit register is:

- (a) $-1024 \leq N \leq +1023$ (b) $-1023 \leq N \leq +1024$
(c) $-512 \leq N \leq +511$ (d) $-511 \leq N \leq +512$
(e) None of the above

SOLUTION

1-bit for the Sign and 9 bits represents the number, so
 $-2^9 \leq N \leq 2^9 - 1$

(c) $-512 \leq N \leq +511$

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Question (2)

Identify the decimal number which is represented next in floating point with the IEEE 754 standard:

11000010100010101100000000000000

- (a) $(-133.375)_{10}$ (b) $(-69.375)_{10}$
(c) $(-138.750)_{10}$ (d) $(-34.6875)_{10}$
(e) $(-8.671875)_{10}$

SOLUTION

<u>1</u>	<u>10000101</u>	<u>000101011000000000000000</u>
S	E	M

2

Question (2) (Cont.)

SOLUTION

1	10000101	000101011100000000000000
S	E	M

✓ Find "real" exponent, n

$$n = E - 127$$

$$= 10000101_2 - 127 = 133 - 127 = 6$$

✓ Put S, M, and n together to form binary result
(Don't forget the implied "1." on the left of the mantissa.)

$$-1.000101011_2 \times 2^6 = (-1000101.011)_2$$

-69

$2^{-2} = 0.25$
 $2^{-3} = 0.125$
 0.375

(b) $(-69.375)_{10}$

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Question (3)

Give the best binary approximation of $A = (26.6)_{10}$
and $B = -(23.4)_{10}$ employing signed **2's-complement representation**
with 2 bits for the fractional part.

SOLUTION

$$A = (26.6)_{10} \approx x = +(11010.10)_2 = (011010.10)_2 = (26.5)_{10}$$

$$\text{or } \approx y = +(11010.11)_2 = (011010.11)_2 = (26.75)_{10}$$

accept $A \approx x$ since $|A-x| < |A-y|$

$$B = -(23.4)_{10} \approx -(010111.10)_2 = -x \text{ with } x = +23.5$$

Note that $-x$ is closer to 23.4 rather than $y=010111.10=23.25$!

If assume that we know how to find the **2's-complement** of an integer
but not of a fractional **negative** number, then we'll consider

- $b = (010111.10)_2 = (010111.10)_2 \times 2^2/2^2 = (01011110)_2/2^2$

- First we'll find the **2's-complement** of $(01011110)_2 = (10100010)_2$

- then we'll divide it by 2^2 to scale back, such that

$$\Rightarrow \text{2's-complement of } (010111.10)_2 = (10100010)_2/2^2 = (101000.10)_2$$

Question (4)

Two 6-bit register contain two numbers $X = 01010$ and $Y = 10101$ (expressed in 2's complement representation). Calculate the sum ($S = X+Y$) and the difference ($D=X-Y$) of these numbers using additions and 2's complementation only. Since both S and D have to be represented with 6 bits, indicate if overflow occurs, and explain how a circuit can detect these situations. Convert in decimal and write each intermediate and final result, to check the correctness of your assertions.

SOLUTION

Since both X AND Y ARE ALREADY IN 2'S COMPLEMENT REPRESENTATION, their value is

$X = (01010)_2 = +10_{10}$ and

$Y = (10101)_2 = -2's \text{ complement of } (0101)_2 = -(1011)_2 = -11_{10}$

$\Rightarrow -11_{10} = -(+11)_{10}$ represented with 6 bits is: $-(001011)_2 = 2's \text{ complement}(001011)_2 = (110101)_2$

As $-Y$ is needed too: $-Y = 2's \text{ complement of } (110101)_2 = (001011)_2 = (\text{just for verification}) = +11_{10}$

	2's complement representation	Base 10 representation		2's complement representation	Base 10 representation																																																																						
$S = X+Y$	<table border="1"> <tr><td>C₆</td><td>C₅</td><td>C₄</td><td>C₃</td><td>C₂</td><td>C₁</td><td>C₀</td></tr> <tr><td>CY:</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>X +</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr> <tr><td>Y</td><td>1</td><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td></tr> <tr><td>S</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr> </table>	C ₆	C ₅	C ₄	C ₃	C ₂	C ₁	C ₀	CY:	0	0	0	0	0	0	X +	0	0	1	0	1	0	Y	1	1	0	1	0	1	S	1	1	1	1	1	1	+10 -11 -1	$D=X-Y$	<table border="1"> <tr><td>C₆</td><td>C₅</td><td>C₄</td><td>C₃</td><td>C₂</td><td>C₁</td><td>C₀</td></tr> <tr><td>CY:</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr> <tr><td>X +</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td></tr> <tr><td>-Y</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td></tr> <tr><td>D</td><td>0</td><td>1</td><td>0</td><td>1</td><td>0</td><td>1</td></tr> </table>	C ₆	C ₅	C ₄	C ₃	C ₂	C ₁	C ₀	CY:	0	0	1	0	1	0	X +	0	0	1	0	1	0	-Y	0	0	1	0	1	1	D	0	1	0	1	0	1	+10 +11 +21
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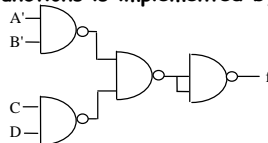
No Overflow

Overflow Detection Expressions

$OFL =$
 if $(SgnX=SgnY) \neq SgnC$, i.e.,
 $(SgnX \cdot SgnY \cdot \overline{SgnC} + \overline{SgnX} \cdot \overline{SgnY} \cdot SgnC)$
 if either of the last two carry bits is 1
 $(C_5 \cdot C_6 + \overline{C_5} \cdot C_6) = C_5 \oplus C_6$

Question (5)

Which of the logic functions is implemented by the following circuit?

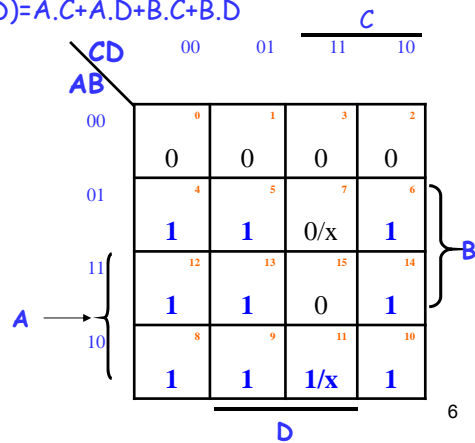


SOLUTION

$F = \overline{A} \cdot \overline{B} \cdot C \cdot D = \overline{A \cdot B} \cdot C \cdot D = (A+B) \cdot (\overline{C+D}) = A \cdot \overline{C} + A \cdot \overline{D} + B \cdot \overline{C} + B \cdot \overline{D}$

$(8,9,12,13) + (8,10,12,14) + (4,5,12,13) + (4,6,12,14) = \Sigma m(4,5,6,8,9,10,11,12,13,14)$

(e) $f(A,B,C,D) = \Sigma m(4,5,6,8,9,10,12,13,14) + X(7,11)$

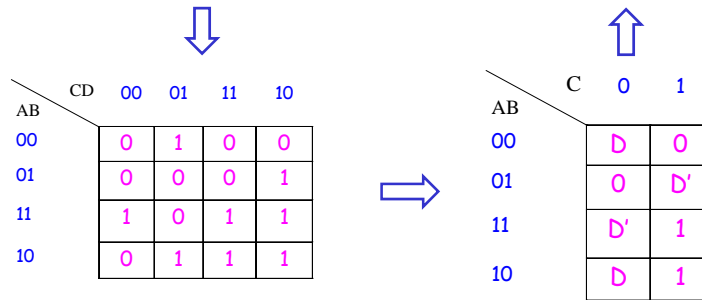
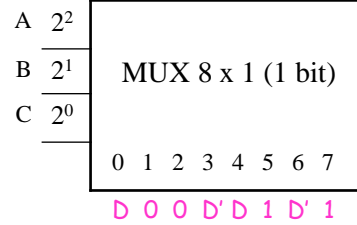


Question (6)

Which of the following logic functions is implemented by the given circuit?

SOLUTION

(e) $f(A,B,C,D) = \Sigma(1,6,9,10,11,12,14,15)$



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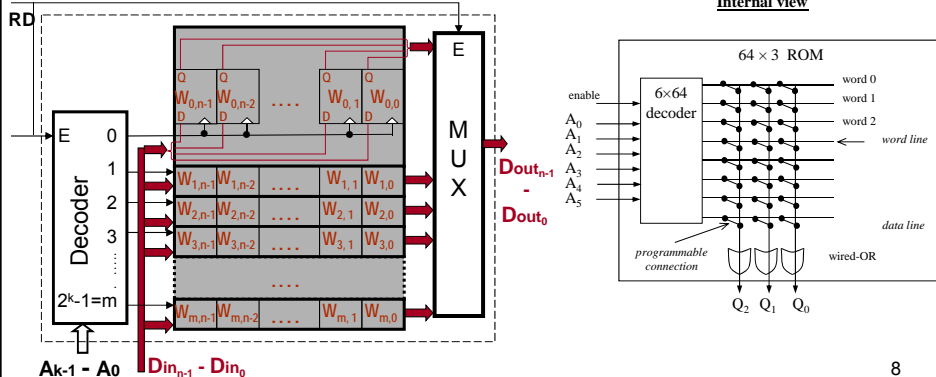
Question (7)

What is the capacity of a ROM, capable to implement three functions of six variables?

SOLUTION

Any combinational circuit of n functions of same k variables can be done with $2^k \times n$ ROM

(d) 64 words of 3 bits



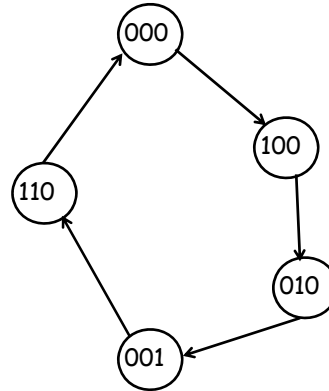
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Question (8)

1) - Draw the state diagram of the sequential circuit whose state table is given below

SOLUTION

A B C	A+B+C'	J _A	K _A	J _B	K _B	J _C	K _C
000	100	1	x	0	x	0	x
001	110	1	x	1	x	x	1
010	001	0	x	x	1	1	x
100	010	x	1	1	x	0	x
110	000	x	1	x	1	0	x



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Question (8) (Cont.)

1) Indicate the correct set of minimized equations of the JK flip-flops inputs

SOLUTION

(c) $J_A = \bar{B}$, $K_A = 1$, $J_B = A+C$, $K_B = 1$, $J_C = \bar{A}.B$, $K_C = 1$.

A B C	A+B+C'	J _A	K _A	J _B	K _B	J _C	K _C
000	100	1	x	0	x	0	x
001	110	1	x	1	x	x	1
010	001	0	x	x	1	1	x
011	x x x	x	x	x	x	x	x
100	010	x	1	1	x	0	x
101	x x x	x	x	x	x	x	x
110	000	x	1	x	1	0	x
111	x x x	x	x	x	x	x	x

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Question (8) (Cont.)

2) If your circuit reaches by mistake any of the 3 states that are not used, determine their corresponding next state.

SOLUTION

(c) $JA = \bar{B}$, $KA = 1$, $JB = A+C$, $KB = 1$, $JC = \bar{A}B$, $KC = 1$.

ABC	A+B+C'
011	000
101	010
111	000

2) Is your circuit auto-corrective?

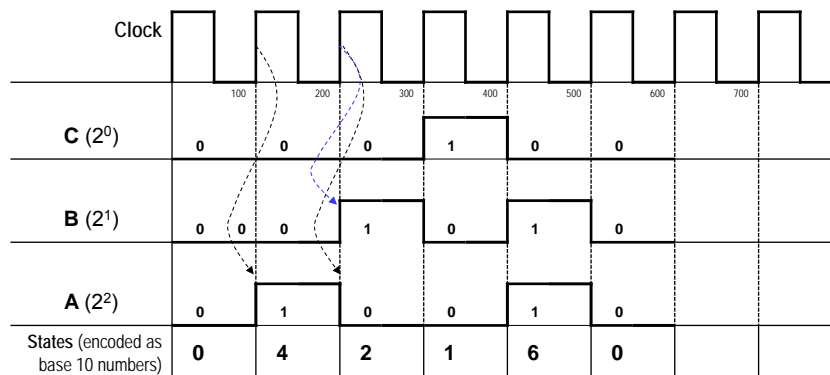
YES, since it returns to a defined state after reaching an undefined state

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Question (8) (Cont.)

3) Draw the time diagram and give the states of the flip-flops' outputs through the first 6 clock pulses, assuming that their initial state (at $t=0$) is 000 and they are triggered on the rising edge of the clock.

SOLUTION



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