

Solution
of Midterm

① (a) $(7 - 3\sqrt{3})^2 = 49 - 42\sqrt{3} + (3\sqrt{3})^2$
 $= 49 - 42\sqrt{3} + 27$
 $= 76 - 42\sqrt{3}$

(b) $4\sqrt{24} - 2\sqrt{12} = 4\sqrt{2^2 \cdot 6} - 2\sqrt{2^2 \cdot 3 \cdot 2}$
 $= 8\sqrt{6} - 12\sqrt{2}$

②

(a) $\frac{5}{3\sqrt{2}} = \frac{5}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{6}$

(b) $\frac{\sqrt{2}}{\sqrt{3} - 3\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{3} - 3\sqrt{2}} \cdot \frac{\sqrt{3} + 3\sqrt{2}}{\sqrt{3} + 3\sqrt{2}}$
 $= \frac{\sqrt{6} + 6}{(\sqrt{3})^2 - (3\sqrt{2})^2}$
 $= \frac{\sqrt{6} + 6}{3 - 18} = -\frac{\sqrt{6} + 6}{15}$

③

(a) $4x(3x^4 - 4x^3 + 6x^2 - 1) - 2x^2(x^2 - 4x + 3)$
 $= 12x^5 - 16x^4 + 24x^3 - 4x - 2x^4 + 8x^3 - 6x^2$
 $= 12x^5 - 18x^4 + 32x^3 - 6x^2 - 4x$

$$\textcircled{d} \quad \frac{3x}{x-4} + \frac{2x}{x+3} = \frac{3x(x+3) + 2x(x-4)}{(x-4)(x+3)}$$

$$= \frac{3x^2 + 9x + 2x^2 - 8x}{(x-4)(x+3)}$$

$$= \frac{5x^2 + x}{(x-4)(x+3)} = \frac{x(x+5)}{(x-4)(x+3)}$$

4

$$\textcircled{a} \quad 3x^2 - 2x - 8 = 3x^2 - 6x + 4x - 8$$

$$= 3x(x-2) + 4(x-2)$$

$$= (x-2)(3x+4)$$

$$\textcircled{b} \quad 64 - 27x^3 = 4^3 - (3x)^3$$

$$= (4-3x)(16 + 12x + 9x^2)$$

prime

$$= (4-3x)(9x^2 + 12x + 16)$$

5

$$\begin{array}{r} -3 \overline{) -4 \ 5 \ 0 \ 8} \\ \underline{ 12 \ -51 \ 153} \\ -4 \ 17 \ -51 \ 161 \end{array}$$

Remainder is
= 161
≠ 0.

Then $(x+3)$ is not a factor of
 $-4x^3 + 5x^2 + 8$

6

$$2(3+2x) = 3(x-4)$$

a

$$6 + 4x = 3x - 12$$

$$x = -18$$

b

$$2x^2 - x - 3 = 0$$

$$2x^2 + 2x - 3x - 3 = 0$$

$$2x(x+1) - 3(x+1) = 0$$

$$(x+1)(2x-3) = 0$$

$$\Rightarrow x = -1 \text{ or } x = \frac{3}{2}$$

c

$$\sqrt{12-x} = x$$

$$12-x = x^2$$

$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$x = -4 \text{ or } x = 3$$

The only solution is $x = 3$

$x = -4$ is impossible solution

7.

(a) $|-x-2| \geq 1$

is equivalent to:

$$-x-2 \geq 1 \quad \text{or} \quad -x-2 \leq -1$$

$$-x \geq 3 \quad \text{or} \quad -x \leq -1$$

$$x \leq -3 \quad \text{or} \quad x \geq -1$$

$$\text{Solution} = (-\infty, -3] \cup [-1, \infty)$$

(b) $x+8 < \frac{1}{2}(x-4)$

$$2x+16 < x-4$$

$$2x-x < -20$$

$$x < -20$$

$$\text{Solution} = (-\infty, -20)$$

8.

(a)

$x=2$ is a vertical line passing by $(2,0)$.

$y=4$ is the only horizontal line perpendicular to $x=2$ and passing by $(3,4)$.

8

$$x^2 + y^2 + 4y - 12 = 0$$

$$x^2 + y^2 + 4y = 12$$

$$x^2 + y^2 + 4y + 4 = 12 + 4$$

$$x^2 + (y+2)^2 = 16$$

This is the equation of a circle with center $(0, -2)$ and radius 4

9

$$f(x) = \sqrt{16-x^2}$$

Condition $16-x^2 \geq 0$

$$(4-x)(4+x) \geq 0$$

	-4	4	
$(4-x)$	+	+	-
$4+x$	-	+	+
$(4-x)(4+x)$	-	+	-

$\Rightarrow 16-x^2$ is positive between -4 and 4

$$\text{Domain} = [-4, 4]$$

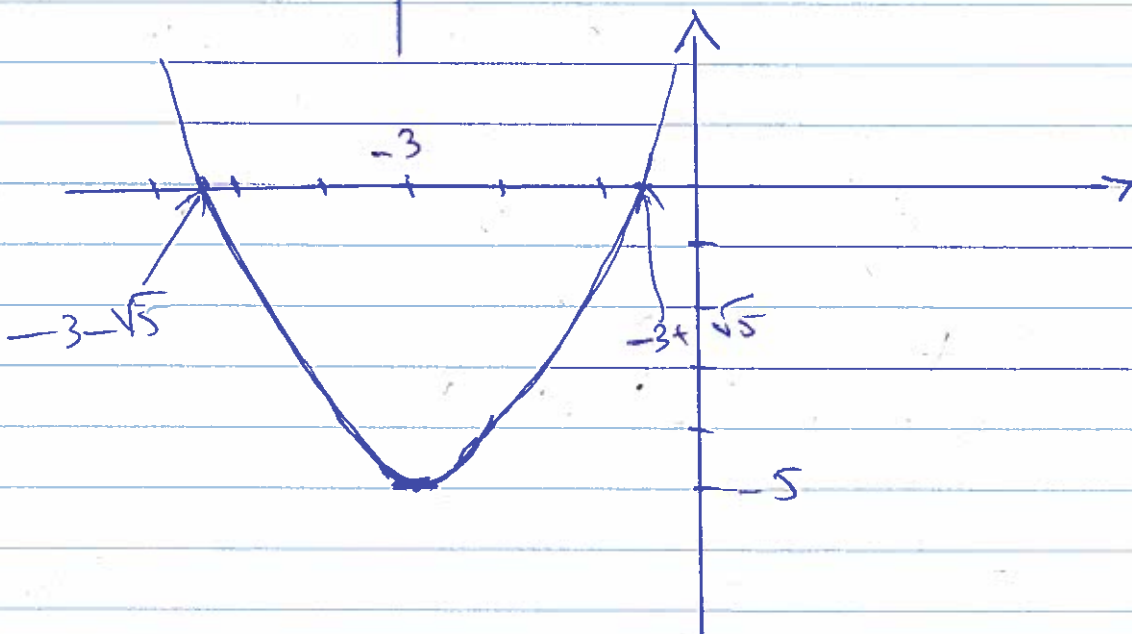
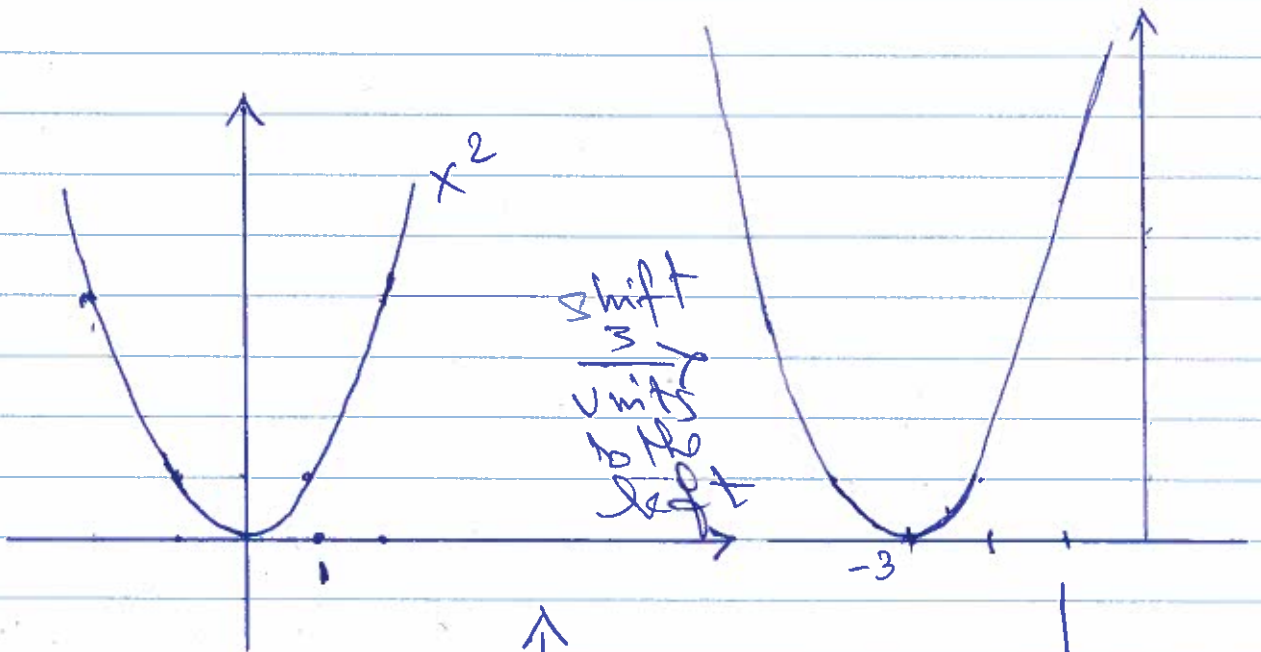
(b)
$$g(x) = \frac{x}{\sqrt{x-3}}$$

Condition: $x-3 > 0$

$x > 3$

Domain = $(3, \infty)$

(10)



vertical shift down by 5 units