Solutions Attached

CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Sections
Mathematics	201	All
Examination	**Date**	**Duration**
Midterm Test	1 November, 2015	1 h 30 min

Special Instructions: Only approved calculators are allowed. **Show all your work for full marks** (indicated in [..])

1. [9] (a) Write the equation of the line which is parallel to the line $2x = 9 + 6y$ and passes through the point $(4, 7)$.

(b) Find the midpoint of the line segment whose endpoints are (-3, 8) and (-5, 4), and calculate the distance of this midpoint from the point (6, -7)

(c) Write the equation of the circle $x^2 + y^2 - 4x = 32$ in the standard form. Find the coordinates of the center and the radius of the circle.

2. [9] Consider the quadratic function $f(x) = \frac{1}{2}x^2 + 3x - 8$.

(a) Express $f(x)$ in standard form.

(b) Find the x- and y- intercepts.

(c) Find its vertex and indicate whether it is the maximum or minimum of f.

3. [8] Given the rational function $f(x) = \dfrac{x^2 - 4}{2x^2 + 5x - 3}$

(a) Write equations of all vertical asymptotes.

(b) Find equations of all horizontal asymptotes (if any).

4. [6] Consider the functions $f(x) = \sqrt{x - 8}$ and $g(x) = \dfrac{1}{x^2 - 9}$.

(a) Find the range and the domain of $f(x)$.

(b) Find the function $g(f(x))$, and determine its domain.

5. [12] Find all solutions of the following equations

(a) $2 \cdot 8^x = 32^{x-1}$

(b) $\log_{10}(2x) + \log_{10}(x - 4) = 1$

(c) $2\log_2(x) = 2 + \log_2(x + 3)$

6. [6] Consider the function $f(x) = \dfrac{2x + 3}{x - 2}$.

(a) Find the inverse function $f^{-1}(x)$.

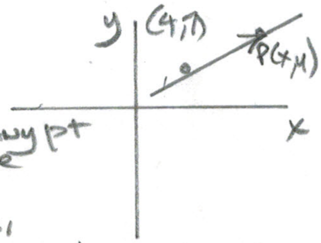
(b) Find the domain and range of $f(x)$ and the domain and range of $f^{-1}(x)$.

Bonus. [3]: If a function $f(x)$ defined for all real x has an inverse $f^{-1}(x)$, does it always follow that also $g(x) = [f(x)]^2$ is invertible? Find $g^{-1}(x)$ if it does, or give an example when $g(x)$ does not have an inverse.

Math 201 Midterm Nov 2015

1 a) ① pt is (4,7) ② Given line $2x = 9 + 6y$

$2x - 9 = 6y$



$y = \frac{2}{6}x - \frac{9}{6}$

$y = \frac{1}{3}x - \frac{3}{2}$ $\Rightarrow$ slope $= \frac{1}{3}$

$\Rightarrow$ slope of Required ∥ line is also $\frac{1}{3}$

③ Eq. of line

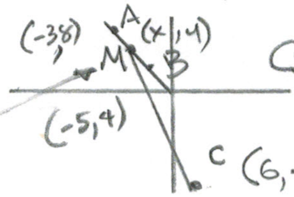
Let P(x,y) be Any pt on line

$M_{AP} = \frac{y_2 - y_1}{x_2 - x_1}$

$\frac{1}{3} = \frac{y-7}{x-4}$ $\Rightarrow$ $3(y-7) = 1(x-4)$

$3y - 21 = x - 4$ $\Rightarrow y = \frac{1}{3}x + \frac{17}{3}$

b)



① let Midpt be M(x,y) $\Rightarrow$ $x = \frac{x_1 + x_2}{2} = \frac{-3 + (-5)}{2} = \frac{-8}{2} = -4$

$y = \frac{y_1 + y_2}{2} = \frac{8+4}{2} = \frac{12}{2} = 6$

$\Rightarrow$ M is pt $(-4,6)$

(-4,6)

② Find distance $MC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(6-[-4])^2 + (-7-6)^2}$

$= \sqrt{10^2 + (-13)^2} = \sqrt{269}$

c) $(x^2 - 4x + 4) + (y^2 - 0) = +32 + 4$

$(x-2)(x-2) + (y-0)(y-0) = 36$

$\sqrt{(x-2)^2 + (y-0)^2} = \sqrt{36}$ $\Rightarrow$ Center is (2,0)

Radius is 6

2a) $y = \frac{1}{2}x^2 + 3x - 8$

$y = \frac{1}{2}(x^2 + 6x + \underline{\underline{9}}) - \frac{8}{1} - \frac{9}{2}$

$y = \frac{1}{2}(x+3)(x+3) - \frac{25}{2}$

$y + \frac{25}{2} = \frac{1}{2}(x+3)^2$ or $2(y - [-\frac{25}{2}]) = (x - [-3])^2$

c) $\Rightarrow$ Vertex is at $(-3, -\frac{25}{2})$ and since multiplier of y is + $\Rightarrow$ opens up

b) x int (let y=0)

$\Rightarrow \frac{1}{2}x^2 + 3x - 8 = 0$

$x^2 + 6x - 16 = 0$

$(x-2)(x+8) = 0$ $\Rightarrow$ x int are $x = 2$, $x = -8$

$x - 2 = 0$	$x + 8 = 0$
$x = +2$	$x = -8$

y int (let x=0)

$y = \frac{1}{2}(0)^2 + 3(0) - 8$

$\Rightarrow$ y int $= -8$

3 a) V.A. let Den = 0 $\Rightarrow 2x^2 + 5x - 3 = 0$

$(2x-1)(x+3) = 0$

$2x - 1 = 0$	$x + 3 = 0$
$x = \frac{1}{2}$	$x = -3$

Check value of Num. when

$x = \frac{1}{2}$	$x = -3$
$(\frac{1}{2})^2 - 4 \neq 0$	$(-3)^2 - 4 \neq 0$
$\Rightarrow x = \frac{1}{2}$ is VA	$\Rightarrow x = -3$ is VA

b) H.A. $\lim_{x \to \infty} \frac{x^2 - 4}{2x^2 + 5x - 3} = \frac{\infty}{\infty}$

$\lim_{x \to \infty} \frac{\frac{x^2}{x^2} - \frac{4}{x^2}}{\frac{2x^2}{x^2} + \frac{5x}{x^2} - \frac{3}{x^2}} = \lim_{x \to \infty} \frac{1 - \frac{4}{x^2}}{2 + \frac{5}{x} - \frac{3}{x^2}} = \frac{1 - 0}{2 + 0 - 0} = \frac{1}{2}$

$\Rightarrow$ HA is $y = \frac{1}{2}$

Note $\lim_{x \to -\infty} \frac{x^2 - 4}{2x^2 + 5x - 3}$ gives Same Answer

4a) $f(x) = \sqrt{x-8}$

① Domain $x \in \mathbb{R} \mid x - 8 \geq 0$ or $[8, \infty)$

② Range $y = \sqrt{x-8}$: RHS is always positive or Exactly 0 (when $x = 8$)

2@3 MARKS

$\Rightarrow$ Range $y \in \mathbb{R} \mid y \geq 0$ or $[0, \infty)$

b) ① $g(x) = \dfrac{1}{x^2 - 9}$

$g(f(x)) = \dfrac{1}{(f(x))^2 - 9} = \dfrac{1}{(\sqrt{x-8})^2 - 9} = \dfrac{1}{x-8-9} = \dfrac{1}{x-17}$

② Domain of $g(f(x))$: $x \in \mathbb{R} \mid x \geq 8$ AND $x \neq 17$

OR $[8, 17) \cup (17, \infty)$

5 a) $2 * 8^x = 32^{x-1}$ | Get bases =

$2^1 * (2^3)^x = (2^5)^{x-1}$

$2^1 * 2^{3x} = 2^{5(x-1)}$

$2^{3x+1} = 2^{5x-5}$

3@4

$\Longrightarrow 3x + 1 = 5x - 5$

$1 + 5 = 5x - 3x$

$2x = 6$

$x = 3$

b) $\log_{10} 2x + \log_{10}(x-4) = 1$

$\log_{10} 2x(x-4) = 1$

$10^1 = 2x(x-4) \Longrightarrow$ $2x^2 - 8x - 10 = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x - 5 = 0$	$x + 1 = 0$
$x = 5$	$x = -1$
	DISCARD (Not when you substitute to check ANS. you can't take log of a negative N.)

c) $2\log_2 x = 2 + \log_2(x+3)$

$\log_2 x^2 - \log_2(x+3) = 2$

$\log_2 \dfrac{x^2}{x+3} = 2$

$2^2 = \dfrac{x^2}{x+3} \Longrightarrow 4(x+3) = x^2$

$x^2 - 4x - 12 = 0$

$(x-6)(x+2) = 0$

$x - 6 = 0$	$x + 2 = 0$
$x = 6$	$x = -2$
	DISCARD

6. a) $y = \dfrac{2x+3}{x-2}$

$x = \dfrac{2y+3}{y-2}$

2@3

$x(y-2) = 2y + 3$

$xy - 2x = 2y + 3 \Longrightarrow xy - 2y = 2x + 3$

$y(x-2) = 2x + 3$

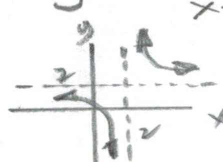
$y = \dfrac{2x+3}{x-2}$

$\Longrightarrow$ if $f(x) = \dfrac{2x+3}{x-2}$ then $f^{-1}(x) = \dfrac{2x+3}{x-2}$

Note: $f(x)$ is its own Inverse.

b) Domain $f(x)$, $f^{-1}(x)$ are = : $x \in \mathbb{R} \mid x \neq 2$

Range $f(x)$, f^{-1} are = : $y \in \mathbb{R} \mid y \neq 2$



Bonus: (It does not follow For Inverse to Exist function must be 1-1) Counter Example { Let $f(x) = x \Rightarrow f$ is 1-1 So f^{-1} exists. If $g(x) = [f(x)]^2 = x^2$ $g(x)$ Not 1-1 $\Rightarrow g(x)$ not Invertible

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