LEC 1
1/11/16

MAT1348 A LEC 1.

Discrete Mathematics:

is about (e.g.):

- logic
- sets
- trees
- graphs

"Discrete" means the elements are separated instead of varying continuously like in calc.

LOGIC:

Example (on a menu):

"... comes with bacon or sausage and toast"

could mean either [bacon or sausage] and toast

or bacon or [sausage and toast]

to a computer.

- From our knowledge of food + menus we know you can get bacon w/ toast OR sausage w/ toast.

Propositional Logic: (Rosen 1.1)

- The aim is to systematize deduction

A proposition is a declarative sentence that makes a claim either true or false.
(at some agreed upon time and place, real or not)

examples. "Today is monday." (True)
"$2+2=3$" (false)
"$2+X=3$" (not a proposition unless x was previously defined)
"for all integers x, $2+x=3$" (FALSE)
"You should get more sleep" (could be a proposition if we are clear what "should" means, otherwise no)
"Eat your veggies!" not declarative ∴ not proposition
"Will you marry me?" not prop.

— Propositions are denoted $p, q, r, s, \dots$

Compound Propositions are made from shorter ones using Logical connectives. ($\neg, \wedge, \vee, \rightarrow, \leftrightarrow$

Example: $p \vee \neg q$ means "p or not q"

Negation $\neg p$ means "not p" or "It's not the case that p is true"

Example: p: "Today is Monday"
$\neg p$: "Today is not monday."

q: "$2x < x^2$ for all real numbers x" (F)
$\neg q$: "It's not the case that $2x < x^2$ for all real numbers

truth table:

q	$\neg q$
T	F
F	T

<u>Conjunction</u>: $p \wedge q$ means "p and q"
(sometimes as "p but q")

truth table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

<u>example</u>: "I took my umbrella, but it didn't rain."

p: "I took my umbrella" $\Rightarrow$ $p \wedge \neg q$
q: "It rained"

<u>Disjunction</u> $p \vee q$ means "p or q"

It's INCLUSIVE or, ie p or q <u>or</u> both

truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Exclusive or: $p \oplus q$ means "p or q but NOT both"

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication (conditional) $p \to q$ means "p implies q" or "If p then q".

Truth table

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

ex. p: I will get elected
q: I will lower taxes
$p \to q$: If I get elected, I will lower taxes

other ways to express $p \to q$:
"p only if q"

"$p \to q$ is not the same as $q \to p$"

T.T.

p	q	$q \to p$
T	T	T
T	F	T
F	T	F
F	F	T

They're not logically equal.

LEC 1

1/11/16

Example: $\underbrace{\text{If it rains}}_{p}$, $\underbrace{\text{then the sidewalk is wet.}}_{q}$

$p \to q$

$q \to p$ "If the sidewalk is wet, then it rains."

other ways to express $p \to q$:

"q if p"

"q when p"

"p is serfficient for q"

Biconditional: $p \leftrightarrow q$ means "if and only if"

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

note:

$p \to q$	$q \to p$
T	T
F	T
T	F
T	T

$p \leftrightarrow q$ is logically the same as "$p \to q$" and "$q \to p$"

$p \leftrightarrow q$ written "iff"

Example: "$\underbrace{\text{If you finish veggies}}_{p}$, $\underbrace{\text{u may have dessert.}}_{q}$"

$p \to q$