CARLETON UNIVERSITY

FINAL EXAMINATION
Fall 2010

DURATION: 3 HOURS

Department Name and Course Number: School of Mathematics and Statistics, MATH1104.
Course Instructors: J. Abdulrahman, A. Sakzad, L. Bourbonnais, and M. Lemire.

AUTHORIZED MEMORANDA

NON-PROGRAMMABLE CALCULATOR PERMITTED

This examination paper **MAY NOT** be taken from the examination room.
Scratch paper is not permitted, you can write on the backs of the pages if needed.

First (Given) Name : **Last Name:**
Student No :

PLEASE READ THE FOLLOWING INSTRUCTIONS BEFORE PROCEEDING

1. Please count your pages now. **This examination has 10 pages**, including this page. If you feel there is a page missing, please report this to your Proctor.
2. **The examination is out of a total of 100 marks** and consists of 17 questions. There are 12 multiple choice questions worth 4 points each and 5 long answer questions. **Place your answers to the multiplce choice questions in the boxes on the second page**. For the long answer questions **you must show all your work**.

Questions	MC 1–12	1	2	3	4	5	Total
Maximum Marks	48	10	10	10	12	10	100
Your mark							

Answers for Multiple Choice Questions:

# 1	# 2	# 3	# 4	# 5	# 6	#7	#8	#9	# 10	# 11	# 12

Part 1. Multiple Choice Questions:

1. Let $u = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$ and $v = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$. What is the angle between the vectors u and v?

   (a) $\pi/3$ **(b)** $-\pi/3$ **(c)** $2\pi/3$ **(d)** $-2\pi/3$ **(e)** $\pi/2$

2. Let A be a 2×2 matrix and $\det A = -3$. What is $\det(2A^2A^TA^{-1})$?

   (a) 36 **(b)** -18 **(c)** 18 **(d)** -36 **(e)** 9

3. Suppose that a system of equations has the augmented matrix $\left[\begin{array}{cc|c} 1 & h & 3 \\ 0 & -5h-10 & k-15 \end{array}\right]$. If the system is inconsistent, then the values of h and k must be:

   (a) $h = -2$, $k = 15$ **(b)** $h = 2$, $k = 15$ **(c)** $h \neq -2$, $k = 15$ **(d)** $h = -2$, $k \neq 15$
   (e) $h \neq -2$, $k \neq 15$

4. Given $A^{-1} = \begin{bmatrix} -1 & 3 \\ 4 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, what is the solution of the equation $A\mathbf{x} = \mathbf{b}$?

(a) $\begin{bmatrix} 5 \\ 9 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 \\ 9 \end{bmatrix}$ **(c)** $\begin{bmatrix} 5 \\ -7 \end{bmatrix}$ **(d)** $\begin{bmatrix} 5 \\ 7 \end{bmatrix}$ **(e)** $\begin{bmatrix} -5 \\ 7 \end{bmatrix}$

5. Let $D = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ be a diagonal matrix and $P = \begin{bmatrix} 1 & 2 \\ -1 & -3 \end{bmatrix}$ be an invertible matrix. What is $(PDP^{-1})^{2010}$?

(a) $\begin{bmatrix} 5 & 4 \\ -6 & -5 \end{bmatrix}$ **(b)** $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **(c)** $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ **(d)** $\begin{bmatrix} -5 & -4 \\ 6 & 5 \end{bmatrix}$ **(e)** $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

6. Let $A = \begin{bmatrix} 5 & -2 \\ 1 & 3 \end{bmatrix}$. Which of the following sets consists of two linearly independent eigenvectors for A?

(a) $\left\{ \begin{bmatrix} 1+i \\ 1 \end{bmatrix}, \begin{bmatrix} 1-i \\ 1 \end{bmatrix} \right\}$ **(b)** $\left\{ \begin{bmatrix} 4+i \\ 1 \end{bmatrix}, \begin{bmatrix} 17 \\ 4-i \end{bmatrix} \right\}$ **(c)** $\left\{ \begin{bmatrix} 4+i \\ 1 \end{bmatrix}, \begin{bmatrix} 4-i \\ 1 \end{bmatrix} \right\}$

(d) $\left\{ \begin{bmatrix} 1+i \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1-i \end{bmatrix} \right\}$ **(e)** $\left\{ \begin{bmatrix} 3+i \\ 1 \end{bmatrix}, \begin{bmatrix} 1+i \\ 1 \end{bmatrix} \right\}$

7. What is the standard form of the complex number $\dfrac{2+4i}{1+i}$?

(a) $1-3i$ **(b)** $3+i$ **(c)** $1+2i$ **(d)** $1-i$ **(e)** $3-i$

8. Let $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 2 & 2 \\ 0 & 0 & 3 & 3 & 3 \\ 0 & 0 & 0 & 4 & 4 \\ 5 & 5 & 5 & 5 & 6 \end{bmatrix}$. What is $\det A$?

(a) 0 **(b)** 6 **(c)** 12 **(d)** 24 **(e)** 48

9. Let $T\colon \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \text{ and } T\left(\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}. \text{ Find } T\left(\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}\right).$$

(a) $\begin{bmatrix} -9 \\ 6 \end{bmatrix}$ **(b)** $\begin{bmatrix} 6 \\ 0 \end{bmatrix}$ **(c)** $\begin{bmatrix} 11 \\ -2 \end{bmatrix}$ **(d)** $\begin{bmatrix} -11 \\ 2 \end{bmatrix}$ **(e)** $\begin{bmatrix} 9 \\ -6 \end{bmatrix}$

10. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 4 \\ 0 & 4 & 2 \end{bmatrix}$. What are the eigenvalues of A?

(a) $1, \ -2, \ 2$ **(b)** $1, \ 2, \ -4$ **(c)** $2, \ 2, \ 4$ **(d)** $0, \ 1, \ 2$ **(e)** $-2, \ 1, \ 6$

11. Find all values of h such that the vectors $\begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ -1 \\ h \end{bmatrix}$ are linearly dependent.

(a) $h = 2$ **(b)** $h \neq 2$ **(c)** $h = -2$ **(d)** $h \neq -2$ **(e)** $h \neq \pm 2$

12. Which of the following sets are subspaces of $\mathbb{R}^3$?

$$U = \left\{ \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \middle| \ a + b = 1 \right\}, \qquad V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \middle| \ a - b = c \right\},$$

$$W = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}, \qquad H = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

(a) U and W **(b)** V and W **(c)** V and H **(d)** U and V **(e)** V only

Part 2: Long Answer Questions. You must show your work!

1. (10 points) Find the general solution of the following linear system:

$$\left\{\begin{array}{rcr} 2x_1 + x_2 - x_3 - x_4 & = & -1 \\ 3x_1 + x_2 + x_3 - 2x_4 & = & -2 \\ -x_1 - x_2 + 2x_3 + x_4 & = & 2 \\ -2x_1 - x_2 \qquad + 2x_4 & = & 3 \end{array}\right.$$

2. (10 points) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

3. (10 points) Use Cramer's rule to find the value of x_3 from the solution to the following linear system:

$$\begin{aligned} x_1 - 2x_2 - 2x_3 &= 7 \\ x_2 + 2x_3 &= 4 \\ x_1 \qquad\quad + x_3 &= 0. \end{aligned}$$

No other method will be accepted.

4. (12 points) Let the matrix A and its reduced row echelon form R be given by the following.

$$A = \begin{bmatrix} 1 & -2 & 2 & 1 & -3 \\ -3 & 1 & -2 & -4 & 0 \\ 4 & -1 & 4 & 7 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & -1 \end{bmatrix} = R.$$

(a) Find a basis for the column space of A.

(b) Find a basis for the row space of A.

(c) Find a basis for the null space of A.

(d) Verify that the dimension of the column space of A plus the dimension of the null space of A is equal to the number of columns of A.

5. (10 points) Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. You are given that the eigenvalues of A are $\lambda_1 = 2$, $\lambda_2 = 2$, and $\lambda_3 = 9$.

   (a) Find a basis of the eigenspace corresponding to each eigenvalue.

   (b) Is A diagonalizable? If so, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$; if not, explain why not.