Sample Midterm Questions (Set 2)

1. Solution: Below are sample answers.

   (a) False. Progress Insurance's success was solely attributed to operational innovation. Progressive did not rely on advertising.

   (b) True. Cross-Docking is pioneered by Walmart.

   (c) False. Taco Bell implemented centralization by decreasing food preparation at each restaurant location.

2. Solution:

   (a) First, we need to compute the capacity rate of each resource: $1/(5+2)$ orders/min for Resource Alpha; $1/(5+10)$ orders/min for Resource Beta; and $1/15$ orders/min for Resource Gamma. Thus, both Beta and Gamma are the bottlenecks, and the capacity rate is $1/15$ orders/min or 4 orders/hr.

   (b) The capacity rate of Alpha is $1/(5+2)$ per minute or $60/7$ per hour. Thus, the implied utilization of Resource Alpha is $4/(60/7) = 0.4667$ or 46.67%.

   (c) Recall from above that the capacity rate is $1/15$ orders/min or 4 orders/hr. Thus, since the profit per unit is $(25\%) \cdot (\$40) = \$10$, the current profit rate is $(\$10) \cdot (4) = 40$ dollars per hour.

   Since two resources are bottlenecks, adding one resource does not increase the overall capacity. So we should *not* lease any resource.

3. Solution:

   (a) If every demand were of Type A, then the amount of each resource required for one unit of the product would have been:

   - Resource Category I: $(10 + 10 + 10)$ minutes (since Type-A demand requires Activities 1, 2A and 3, all of which require Resource Category I).
   - Resource Category II: 0 minutes (Type-A demand requires Activities 1, 2A and 3, but none of these activities require Resource Category II).

   If every demand were of Type B, then the amount of each resource required for one unit of the product would have been:

   - Resource Category I: $(10 + 0 + 10)$ minutes (since Type-A demand requires Activities 1, 2B and 3, out of which, only Activities 1 and 3 require Resource Category I)
   - Resource Category II: 30 minutes (since Type-A demand requires Activities 1, 2B and 3, out of which, only Activity 2B requires Resource Category II).

   Consider a product mix consisting of $\frac{1}{5}$ units of A and $\frac{4}{5}$ units of B. Then, the amount of each resource required for one unit of the product (for the product mix) is as follows:

   - Resource Category I: $\frac{1}{5}(10 + 0 + 10) + \frac{4}{5}(10 + 0 + 10) = \left[10 + \frac{1}{5} \cdot (10) + 10\right]$ minutes
   - Resource Category II: $\frac{1}{5}(0) + \frac{4}{5}(30) = \left[\frac{4}{5} \cdot (30)\right]$ minutes.

   Thus, the capacity rate for each resource is given as follows:

   - Resource Category I: $1/\left[10 + \frac{1}{5} \cdot (10) + 10\right] = 0.04545$ orders/min.
   - Resource Category II: $1/\left[\frac{4}{5} \cdot (30)\right] = 0.04167$ orders/min per resource. But there are 2 units of Category II; thus, the capacity rate of this resource pool is $2 \cdot 0.04167 = 0.08333$ orders/min.

   Thus, the bottleneck resource is Category I.

   (Note: one could have computed capacity rates in terms of *orders per hour* instead of *orders per minute*.)

   (b) The *input rate* is 2.5 customers per hour, or 0.04167 customers per minute. Thus the *implied* utilization rates are:

   - Resource Category I: $0.04167/0.04545 = 91.67\%$

- Resource Category II: $0.04167/0.8333 = 50\%$

(c) Note that the input rate is smaller than the capacity rate; thus, the throughput rate is the same as the input rate. The current average rate per hour is:

$$\frac{1}{5} \cdot (2.5) \cdot \$100 + \frac{4}{5} \cdot (2.5) \cdot \$110 \;=\; \$270.$$

(d) Option 1: Increase demand rate of Type A (from 0.50 per hour to 0.55 per hour). In this case, even if every additional type A customer can be accommodated (resulting in 0.05 more of Type A), the additional revenue would be only $0.05 \cdot \$110 = 5.5$ dollars (much less than the marketing cost).

Option 2: Increase demand rate of Type B (from 2 to 2.2 per hour). In this case, even if every additional type B customer can be accommodated (resulting in 0.2 more of Type B), the additional revenue would be $0.2 \cdot \$100 = 20$ dollars (much less than the marketing cost).

Thus, it does not make sense to spend more than \$20 to increase demand.

4. SOLUTION: It is helpful to draw the inventory build-up diagram.

- Initially (at 8AM), there is no "inventory of customers".
- Between 8AM and 9AM, there is no inventory of customers.
- Between 9AM and 12 noon, the inventory of customers increases at the rate of $30 - 10 = 20$ per hour, reaching 60 at 12 noon.
- Between 12 noon and 1 PM, the inventory decreases at the rate of $20 - 10 = 10$ per hour, reaching 50 at 1PM.
- Between 1PM and 3PM, the inventory increases at the rate of $45 - 40 = 5$ per hour, reaching 60 at 3PM.

(a) From the above description of the inventory build-up diagram, there are 50 people waiting at 11:30AM.

Furthermore, there are 60 people at 3PM, who are sent away.

(b) Consider the inventory build-up diagram described above. To compute the area under the "curve", we divide the region as follows:

- 8AM-9AM: 0
- 9AM-12 noon: Area $= 3 \cdot 60/2 = 90$
- 12 noon-1PM: Area $= (60 + 50)/2 = 55$
- 1PM-3PM: Area $= 2\cot(50 + 60)/2 = 110$

Total area = 255. Dividing it by 7 hours (8AM to 3PM), the average number of customers in the system is $255/7 = 36.43$.

The total number of customers who showed up is $30 \cdot (5 \text{ hours}) + 45 \cdot (2 \text{ hours}) = 240$. And every customer who showed up leaves the system – whether he/she is served or not. Thus,

$$\begin{aligned}
\text{“Throughput” Rate} &= \frac{240 \text{ customers}}{7 \text{ hours}} \\
&= 34.29 \text{ customers/hour} \\
\text{Average Flow Time} &= \frac{\text{Average Inventory}}{\text{“Throughput” Rate}} = \frac{36.43 \text{ customers}}{34.29 \text{ customers/hour}} \\
&= 1.06 \text{ hour} \;=\; 64 \text{ minutes}
\end{aligned}$$

5. SOLUTION: We can model this with an $M/M/1$ queue with $\lambda = 15$ per hour and $\mu = 60/3 = 20$ per hour.

(a) Utilization $\rho = \lambda/\mu = 15/20 = 75\%$

(b) $I_q = \rho^2/(1 - \rho) = (0.75)^2/(1 - 0.75) = 2.25$

(c) $I = I_q + I_s = I_q + \rho = 2.25 + 0.75 = 3.0$.

(d) By Little's Law, $T_q = I_q/\lambda = 2.25/15 = 0.15$ hour or 9 minutes.

(e) By Little's Law, $T = I/\lambda = 3/15 = 0.2$ hour or 12 minutes.

(f) Using the CDF of exponential distribution, $P[a > 1/12 \text{ hr}] = 1 - P[a \leq 1/12 \text{ hr}] = \exp(-\lambda \cdot \frac{1}{12}) =$ 28.7%.

(g) With the change, we model the system as an $M/D/1$ queue. As a result, both I_q and T_q become halved, i.e., $I_q = 1.125$ and $T_q = 0.075$ hour (or 4.5 minutes). Furthermore, $I = 1.125 + 0.75 = 1.875$ and $T = 0.125$ hour (or 7.5 minutes). Other answers remain unchanged.

6. SOLUTION:

(a) There are four paths: A-B-D-E-G, A-B-D-F-G, A-C-D-E-G and A-C-D-F-G. The longest path is A-C-D-F-G, and it takes 25 weeks.

(b) Construct the following information regarding crashing.

Activity	Cost per week ($)	Time (weeks)
A	1,000	1
B	2,000	1
C	1,200	1
D	1,500	1
E	1,000	1
F	1,500	2
G	3,000	1

- Initially the critical path is A-C-D-F-G. Reduce Activity A by 1 week at the additional cost of $1,000.
- Critical path: A-C-D-F-G. Reduce Activity C by 1 week at the additional cost of $1,200.
- Critical path: A-C-D-F-G and A-B-D-F-G. Reduce Activity D (or F) by 1 week at the additional cost of $1,500.
- Critical path: A-C-D-F-G and A-B-D-F-G. Reduce Activity F (or D) by 1 week at the additional cost of $1,500.

Thus, the costs of shortening the project is $5,200.