Carleton University

Department of Mathematics and Statistics

MATH 1119C: Linear Algebra with Applications to Business and Economics

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Practice Test # 1 Solutions – Version A

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Surname ____________________ First Name ____________________

Student # ____________________

Instructions:

- You have 50 minutes to complete this test.
- The number of points available for each question is indicated in square brackets.
- You must justify your answers to receive full marks on the long answer questions.
- All work to be considered for grading should be written in the space provided. The reverse side of pages is for scrap work. If you find that you need extra space in order to answer a particular question, you should continue on the reverse side of the page and indicate this *clearly*. Otherwise, the work written on the reverse side of pages will not be considered for marks.
- Write your student number at the top of each page in the space provided.
- No notes, books, or outside scrap paper (the last page of the exam is for scrap, and you can use the reverse side of the other exam pages).
- Non-programmable calculators are permitted.
- You should write in *pen*, not pencil.
- You may use the last page of the exam as scrap paper.

Good luck!

Please do not write in the table below.

Question	1	2	3	4	5	6	Total
Maximum	1	1	1	3	6	5	17
Grade							

PART 1: Multiple Choice & True/False

QUESTION 1. [**1 point**] A linear system whose augmented matrix has 8 rows and 11 columns has how many equations and how many variables?

a) 8 equations and 10 variables
b) 8 equations and 11 variables
c) 11 equations and 8 variables
d) 11 equations and 7 variables

Solution: a.

QUESTION 2. [**1 point**] Evaluate the following product.

$$\begin{bmatrix} 3 & -2 \\ 9 & 0 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -6 \end{bmatrix}$$

a) $\begin{bmatrix} 1 \\ 9 \\ -2 \end{bmatrix}$ b) $\begin{bmatrix} 15 \\ 9 \\ 6 \end{bmatrix}$

c) $\begin{bmatrix} -20 \\ -54 \\ 5 \end{bmatrix}$ d) $\begin{bmatrix} 9 \\ 9 \\ -5 \end{bmatrix}$

Solution: b. You can see this by using the rule for matrix-vector multiplication covered in class:

$$\begin{aligned}
\begin{bmatrix} 3 & -2 \\ 9 & 0 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ -6 \end{bmatrix} &= (1) \begin{bmatrix} 3 \\ 9 \\ -1 \end{bmatrix} + (-6) \begin{bmatrix} -2 \\ 0 \\ -1 \end{bmatrix} \\
&= \begin{bmatrix} (1)(3) \\ (1)(9) \\ (1)(-1) \end{bmatrix} + \begin{bmatrix} (-6)(-2) \\ (-6)(0) \\ (-6)(-1) \end{bmatrix} \\
&= \begin{bmatrix} 3 \\ 9 \\ -1 \end{bmatrix} + \begin{bmatrix} 12 \\ 0 \\ 6 \end{bmatrix} \\
&= \begin{bmatrix} 3+12 \\ 9+0 \\ (-1)+6 \end{bmatrix} \\
&= \begin{bmatrix} 15 \\ 9 \\ 5 \end{bmatrix}.
\end{aligned}$$

QUESTION 3. [**1 point**] For which of the following values of x and y is the matrix below in echelon form?

$$\begin{bmatrix} 2 & -1 & 0 \\ x & y & 3 \end{bmatrix}$$

a) $x = 1, y = 0$
b) $x = 1, y = 1$
c) $x = 0, y = 0$
d) $x = 0, y = 1$
e) both c) and d)

Solution: e.

QUESTION 4. [**3 marks**] True or false.

F It is possible for a linear system to have exactly 10 different solutions.

T It is not possible for a linear system with 4 equations and 5 variables to have a unique solution.

T If a linear system of equations with variables $x_1, x_2, \ldots, x_n$ is homogeneous, then $x_1 = 0, x_2 = 0, \ldots, x_n = 0$ must be a solution.

PART 2: Long Answer Be sure to justify your answers by showing your work.

QUESTION 5. [**6 points**] Consider the system of equations

$$\begin{array}{rcrcrcr} x_1 & - & 5x_2 & - & 2x_3 & = & 2 \\ -x_1 & + & 8x_2 & + & 5x_3 & = & -11 \\ -2x_1 & + & 7x_2 & & & = & 3 \end{array}$$

a) Find the augmented matrix for the system.
b) Row reduce the augmented matrix to reduced echelon form.
c) Write down all solutions to the system based on your reduced matrix.

Solution:

a) The augmented matrix is

$$\begin{bmatrix} 1 & -5 & -2 & 2 \\ -1 & 8 & 5 & -11 \\ -2 & 7 & 0 & 3 \end{bmatrix}.$$

b)

$$\begin{bmatrix} 1 & -5 & -2 & 2 \\ -1 & 8 & 5 & -11 \\ -2 & 7 & 0 & 3 \end{bmatrix} \xrightarrow{R_2=R_2+R_1} \begin{bmatrix} 1 & -5 & -2 & 2 \\ 0 & 3 & 3 & -9 \\ -2 & 7 & 0 & 3 \end{bmatrix} \xrightarrow{R_3=R_3+2R_1}$$

$$\begin{bmatrix} 1 & -5 & -2 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & -3 & -4 & 7 \end{bmatrix} \xrightarrow{R_3=R_3+R_2} \begin{bmatrix} 1 & -5 & -2 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_3=(-1)R_3}$$

$$\begin{bmatrix} 1 & -5 & -2 & 2 \\ 0 & 3 & 3 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2=R_2+(-3)R_3} \begin{bmatrix} 1 & -5 & -2 & 2 \\ 0 & 3 & 0 & -15 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1=R_1+2R_3}$$

$$\begin{bmatrix} 1 & -5 & 0 & 6 \\ 0 & 3 & 0 & -15 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_2=(\frac{1}{3})R_2} \begin{bmatrix} 1 & -5 & 0 & 6 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{R_1=R_1+5R_2}$$

$$\begin{bmatrix} 1 & 0 & 0 & -19 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

c) The linear system equivalent to the reduced echelon form in part b) is

$$\begin{array}{rcr} x_1 & = & -19 \\ x_2 & = & -5 \\ x_3 & = & 2 \end{array}$$

So, the *only* solution to the system is

$$(x_1, x_2, x_3) = (-19, -5, 2).$$

QUESTION 6. [**5 points**] Write down the general description of the solutions to the linear system below. (*Hint*: Notice that the augmented matrix for the system is already in echelon form)

$$\begin{array}{rcrcrcr} x_1 & - & 5x_2 & + & 10x_3 & = & -4 \\ & & x_2 & & & = & 2 \end{array}$$

Solution: You could complete this by forming the augmented matrix, reducing to reduced echelon form, and then writing out the solutions, but you can also do this by back-substitution, which is probably easier. The last equation says $x_2 = 2$, so the system is equivalent to

$$\begin{array}{rcrcrcr} x_1 & - & 5(2) & + & 10x_3 & = & -4 \\ & & x_2 & & & = & 2 \end{array}$$

which can be rearranged to

$$\begin{array}{rcrcrcr} x_1 & & & + & 10x_3 & = & -4+10 \\ & & x_2 & & & = & 2 \end{array}$$

The general description of the solution set comes by solving the first equation for the basic variable x_1:

$$\begin{array}{rcl} x_1 & = & 6 - 10x_3 \\ x_2 & = & 2 \\ x_3 & = & \text{free} \end{array}$$