Answer Key , Assignment 1

1. (a) FALSE. Utility functions only tell us how the consumer ranks consumption bundles. So if Luke's utility increases from 50 to 100, we can only say he is now better off, but not that he is *twice* as well-off.

   (b) FALSE. A consumer chooses the bundle to maximize her utility, not her *marginal* utility.

   (c) TRUE. The best way to answer this question is to draw a graph, showing that the new optimal choice lies on a lower indifference curve than the original optimal choice. Note that Connie's preferences are perfect complements, therefore her indifference curves are "L-"shaped. Her optimal choices always lie at the kink points of the "L-"shapes.

   (d) TRUE. Luxury goods are goods such that consumption of the good increases by a higher percentage than income does. When the consumer has perfect complements preferences, her consumption of either good increases proportionally with income. In other words, her preferences are homothetic. Therefore, neither good can be a luxury good.

2. a. Observe that $v(x_1, x_2) = \ln u(x_1, x_2)$. From 2(a), $f(u) = \ln u$ is a monotonic transformation. Monotonic transformations of a utility function represent the same preferences.

   b. Use the optimum condition

$$MRS = \text{slope of budget line},$$

and the budget constraint to derive the optimal consumption bundle. In this case,

$$MRS = -\frac{MU_1}{MU_2} = -\frac{x_2}{x_1}.$$

The slope of the budget line is

$$-\frac{p_1}{p_2} = -\frac{20}{10} = -2.$$

Therefore, the optimal consumption bundle (x_1, x_2) satisfies

$$-\frac{x_2}{x_1} = -2.$$

So, $x_2 = 2x_1$. Substituting this into the budget equation

$$20x_1 + 10x_2 = 200,$$

we get $x_1 = 5$. Since $x_2 = 2x_1$, $x_2 = 10$. So, her optimal consumption bundle is $(x_1, x_2) = (5, 10)$. The graph is omitted here.

c. Now the price the consumer has to pay for CDs is

$$p_1' = 20(1 + 25\%) = 25.$$

The slope of the budget line becomes

$$-\frac{p_1'}{p_2} = -\frac{25}{10} = -\frac{5}{2}.$$

Therefore, the new optimal choice (x_1', x_2') satisfies

$$-\frac{x_2'}{x_1'} = -\frac{5}{2}.$$

Using this in the new budget line

$$25x_1 + 10x_2 = 200,$$

we get $x_1' = 4$, and $x_2' = 10$. Therefore, the consumer's new optimal consumption bundle is $(4, 10)$.

d. The consumer is worse off. Her utility is 50 before the tax and 40 after the tax. It confirms the answer to 3(c) in Assignment 1. That is, the consumer cannot become better off.

3. (a) The indifference curve will be the NE boundary of the two lines . (See Figure)

   (b) The slope of the budget line is $-\frac{p_1}{p_2}$. If the budget line is steeper than 2, then $x_1 = 0$. This means, $x_1 = 0$ if $\frac{p_1}{p_2} > 2$.

   (c) By an identical logic, $x_2 = 0$ if $\frac{p_1}{p_2} < \frac{1}{2}$.

   (d) If the optimum is unique and on the interior, then it must be, $2x_1 + x_2 = x_1 + 2x_2$ or $x_1 = x_2$

4. (a) The consumer has quasilinear preferences. The indifference curves are vertical translations of each other.

(b) Use the optimum condition

$$MRS = \text{slope of budget line},$$

and the budget constraint to derive the optimal consumption bundle. In this case,

$$MRS = -\frac{MU_1}{MU_2} = -\frac{1/x_1}{1} = -\frac{1}{x_1}.$$

The slope of the budget line is

$$-\frac{p_1}{p_2} = -\frac{5}{1} = -5.$$

Therefore, the optimal consumption bundle (x_1, x_2) satisfies

$$-\frac{1}{x_1} = -5.$$

So, $x_1 = 1/5$. Substituting this into the budget equation

$$5x_1 + x_2 = 20,$$

we get $x_2 = 19$. So, her optimal consumption bundle is $(x_1, x_2) = (1/5, 19)$.

(c) Note we can solve the optimal choice of x_1 by using the MRS condition. Since neither price has changed, under the new income, $x_1' = 1/5$ as before. Using the budget line

$$5x_1 + x_2 = 30,$$

we have $x_2' = 29$. Therefore, the consumer's new optimal consumption bundle is $(1/5, 29)$. Note that her optimal choice of good 1 does not depend on income.

(d) If the price of milk reduces to $p_1' = 3$. The optimal consumption of milk is $x_1' = 1/3$. Hence the optimal consumption bundle is $(x_1', x_2') = (1/3, 19)$

(e) As mentioned in part (c), the optimal choice of good 1 does not depend on income. Hence the income effect will be 0. Therefore the entire change in the consumption of commodity 1 (milk) is the substitution effect which is equal to $(1/3 - 1/5) = 2/15$.

5. (a) Use the optimum tangency condition

$$MRS = \text{slope of budget line},$$

and the budget constraint to derive the optimal consumption bundle. In this case,

$$MRS = -\frac{MU_x}{MU_y} = -\frac{2/x}{1/y} = -\frac{2y}{x}.$$

The slope of the budget line is

$$-\frac{p_x}{p_y} = -\frac{2}{1} = -2.$$

Therefore, the optimal consumption bundle (x, y) satisfies

$$-\frac{2y}{x} = -2.$$

So, $y = x$. Substituting this into the budget equation

$$2x + y = 18,$$

we get $x = y = 6$. So, her optimal consumption bundle is $(x*, y*) = (6, 6)$. The graph is omitted here.

(b) Now the price the consumer has to pay for good x is

$$p'_x = 3.$$

The slope of the budget line becomes

$$-\frac{p'_x}{p_y} = -\frac{3}{1} = -3.$$

Therefore, the new optimal choice (x'', y'') satisfies

$$-\frac{2y''}{x''} = -3.$$

Using this in the new budget line

$$3x + y = 18,$$

we get the new optimal choices are $x'' = 4$ and $y'' = 6$. Therefore, the consumer's new optimal consumption bundle is $(4, 6)$.

(c) The income that corresponds to the pivoted budget line is

$$m' = p'_x x^* + p_y y^* = 3 \times 6 + 1 \times 6 = 24,$$

the income that under new prices make the old consumption bundle (x^*, y^*) affordable to the consumer. The budget equation that corresponds the pivoted budget line is

$$3x + y = 24.$$

Using the optimum condition

$$-\frac{2y''}{x''} = -3,$$

and this budget equation, we get the consumer's optimal choices are $x' = \frac{16}{3}$ and $y' = 8$. Thus the substitution effect is

$$x' - x^* = \frac{16}{3} - 6 = -\frac{2}{3},$$

and the income effect

$$x'' - x' = 4 - \frac{16}{3} = -\frac{4}{3}.$$

(d) Observe that $u(x, y) = \ln x^2 y$. Thus, it is obtained by applying the monotonic transformation $f(v) = \ln v$ to a typical Cobb-Douglas utility function $v(x, y) = x^2 y$. Monotonic transformations of a utility function represent the same preferences. Hence, the consumer has Cobb-Douglas preferences.