Faculty of Science »
Department of mathematics and statistics

Final Exam - Solutions
MAT 2377
Winter 2012

Short Answer Questions

[4] 1.

(a) The probability density function of X is

$$f(x) = F'(x) = \frac{1}{10}\, e^{-(x-20)/10}, \quad x > 20.$$

(b)

$$P(30 < X < 40) = F(40) - F(30) = e^{-1} - e^{-2} = 0.2325$$

and

$$P(X > 30) = 1 - F(30) = e^{-1} = 0.3679.$$

(c) We want

$$\begin{aligned} P(X < 40 | X > 30) &= \frac{P(\{X < 40\} \cap \{X > 30\})}{P(X > 30)} \\ &= \frac{P(30 < X < 40)}{P(X > 30)} = \frac{0.2325}{0.3679} = 0.6320 \end{aligned}$$

[4] 2. Let X be the number of twisted plates among 1000 plates produced by this machine. X has a binomial distribution with $n = 1000$ and $p = 1/240 = 0.004167$.

(a) The expected number of twisted plates among 1000 plates produced by this machine is $E[X] = n\,p = 4.167$.

(b) Let T be the number of plates produced until the first twisted plate occurs. T has a geometric distribution with $p = 1/240 = 0.004167$. We want

$$V[T] = \frac{1-p}{p^2} = 57,360.$$

(c) Let Y be the number of twisted plates among $n = 500$. Y has a binomial distribution with $n = 500$ and $p = 1/240 = 0.004167$. We want

$$P(Y \le 2) = \sum_{k=0}^{2} \binom{500}{k} (1/240)^k\, (1 - 1/240)^{500-k} = 0.6541.$$

Alternatively, we could approximate this probability with a Poisson approximation with $\lambda = n\,p = 2.0833$ since n is large and p is small. The approximation is

$$P(Y \leq 2) \approx e^{-2.0833}\left[\frac{2.0833^0}{0!} + \frac{2.0833^1}{1!} + \frac{2.0833^2}{2!}\right] = 0.6541.$$

[4] 3.

(a) Let $\overline{X}$ be the sample mean for the life of $n = 25$ tires. $\overline{X}$ has a normal distribution with a mean of $\mu = 58,915$ km and a standard deviation of $\sigma/\sqrt{n} = 3,545/\sqrt{25} = 709$ km. We want

$$P(\overline{X} > 61,000) = 1 - \Phi\left(\frac{61,000 - 58915}{709}\right) = 1 - \Phi(2.94) = 0.0016.$$

(b) Let X be the life of a tire. X has a normal distribution with a mean of $\mu = 58,915$ km and a standard deviation of $\sigma = 3,545$ km. We want x such that

$$0.9 = P(X < x) = \Phi\left(\frac{x - 58,915}{3,545}\right).$$

From the table for the standard normal, we get

$$1.28 = \frac{x - 58,915}{3,545} \quad \Rightarrow \quad x = 1.28\,(3,545) + 58,915 = 63,452.60$$

This means that 90% of the tires of a life that is less than 63,452.60 km.

(c) A 95% confidence interval for the mean life (in km) is

$$\overline{x} \pm t_{0.025;15}\frac{s}{\sqrt{n}} = 60,140.0 \pm 2.131\left(\frac{3,655.45}{\sqrt{16}}\right) = [58,192.56; 62,087.44].$$

(d) We want to test $H_0 : \mu = 58,915$ km against $H_1 : \mu > 58,915$ km. Under the conditions of a normal population with σ unknown, we will use a t-test statistic. Its observed value is

$$t_0 = \frac{60,140.0 - 58,915}{3,655.45/\sqrt{16}} = 1.34.$$

It is a right sided alternative the p-value is $p = P(T > 1.34)$, where T has a t distribution with $\nu = 15$ degrees of freedom. From the table for the t distribution, we get

$$0.05 < p\text{-value} < 0.10.$$

At a level of significance of 5%, we could not conclude that the mean has increased. However, at a level of significance of 10%, we could conclude that the mean has increased.

[4] 4.

(a) Let X be the number of bugs in 500 lines of code. X has a Poisson distribution with a mean of $\lambda = 3.6(0.5) = 1.8$ bugs. We want

$$P(X \geq 2) = 1 - P(X \leq 1) = 1 - e^{-1.8}\left[\frac{1.8^0}{0!} + \frac{1.8^1}{1!}\right] = 0.5372.$$

(b) Let Y be the number of bugs in 10,000 lines of code. X has a Poisson distribution with a mean of $\lambda = 3.6(10) = 36$ bugs. We want $\sigma_Y = \sqrt{\mathsf{Var}[Y]} = \sqrt{\lambda} = 6$ bugs.

(c) We can model the bugs and typos as a Poisson process with a rate of $4.2(5) + 3.6 = 24.6$ problems per 1000 lines. Let W be the number of problems in 49 lines of code. W has a Poisson distribution with a mean of $\lambda = (24.6)(0.049) = 1.2054$ problems. The probability that the compiler stops before the 50th line of code is

$$P(W \geq 1) = 1 - P(W = 0) = 1 - e^{-1.2054} = 0.7004.$$

Multiple Choice Questions

1-E
2-A
3-D
4-C
5-C
6-B
7-D
8-A
9-B
10-A
11-E
12-E
13-A
14-B