Université d'Ottawa · University of Ottawa
Faculté des sciences
Mathématiques et de statistique
Faculty of Science
Mathematics and Statistics

Final Exam for MAT2377
Probability and Statistics for Engineers.

Time : 3 hours **Professor : G. Lamothe**

Name : ______________________________

Student Number : ______________________________

Calculators are permitted. It is an open book exam. Textbook and notes are permitted.
There are 4 short answer questions and 14 multiple choice questions.
The exam will be marked on a total of 30 points.

Submit your answers for the multiple choice questions in the following table.

Question	Answer	Question	Answer
1		8	
2		9	
3		10	
4		11	
5		12	
6		13	
7		14	

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Short Answer Questions

[**4**] 1. Let X be a continuous random variable with the following cumulative distribution function

$$F(x) = 1 - e^{-(x-20)/10}, \quad x > 20.$$

(a) Give the probability density function of X.

(b) Compute $P(30 < X < 40)$ and $P(X > 30)$.

(c) Given that the random variable is larger than 30, compute the conditional probability that it is smaller than 40.

[**4**] 2. A machine cuts large sheets of metal into smaller plates independently. The cutting process occasionally makes twisted plates that are flattened before they go to the next stage of processing. On average, 1 out of every 240 plates is twisted.

(a) Find the expected number of twisted plates among 1000 plates produced by this machine.

(b) What is the variance of the number of plates produced until the first twisted plate occurs.

(c) Find the probability that there are 2 twisted plates or less in 500 produced.

[4] 3.

A tire manufacturer has been using a particular rubber compound for years. The life of tire manufactured with this compound (in km) is normally distributed with a mean of $58,915$ km and a standard deviation of $3,545$ km.

(a) If we select $n = 25$ tires at random, what is the probability that the mean life of the tires in this sample will be more than $61,000$ km.

(b) Give the 90th percentile for the life of a tire (in km). In words, give an interpretation to this value within the context of the question.

(c) A research engineer working for this manufacturer is investigating tire life for a new rubber compound and has built 16 tires. The sample mean and standard deviation are $60,140.0$ and $3,655.45$. Build a 95% confidence interval for the mean life of a tire manufactured with the new compound. You can assume that the life of a tire (under the new compound) is normally distributed. However, you can not assume that you know the standard deviation of the life.

(d) Compute the p-value to verify the hypothesis that the new compound does improve the life of a tire. Based on this p-value, can the engineer conclude that the new compound has increased the mean life of a tire ?

(Question 3 cont.)

(Question 3 cont.)

[4] 4. A software engineer writes a long and complex computer program that dictates the behaviour of an autonomous mining robot. The program features bugs (mistakes in the logic of the algorithm) according to a Poisson process, with an average of 3.6 bugs per 1000 lines of code.

(a) Find the probability that there are at least 2 bugs in the first 500 lines of code.

(b) What is the standard deviation of the number of bugs in 10,000 lines of code?

(c) Independently of the bugs, the engineer also occasionally makes a typo, according to a Poisson process with mean 4.2 per 200 lines of code. Both bugs and typos will cause the compiler to stop at the line of the code where there is a problem. Find the probability that the compiler stops before the 50th line of code.

(Question 4 cont.)

Multiple Choice Questions

Submit your answers for the multiple choice questions in the table found on the front page.

[**1**] 1. Strands of copper wire from a manufacturer are analyzed for strength and conductivity. The probability that its conductivity is high is 82%. The probability that its strength is high is 75%. The probability that its strength is high and its conductivity is not high is 23%. What is the probability that its conductivity is high and its strength is high?

(A) 0.1725 (B) 0.615 (C) 0.86 (D) 0.59 (E) 0.52

[**1**] 2. Errors in an experimental transmission channel are found when the transmission is checked by a certifier that detect missing pulses. The number of errors in a byte is a random variable with the following probability mass function :

x	0	1	4	7
$f(x)$	0.1	0.4	0.3	0.2

When there is at least one error in the transmission, what is the conditional probability that the number of errors is 7?

(A) 2/9 (B) 2/10 (C) 2/5 (D) 2/7 (E) 1

[**1**] 3. The table below gives the summary statistics for the anticipated time to completion (in months) and actual time to completion of construction projects of 18 high-rise condo towers. Let μ_1 be the mean of actual time to completion and μ_2 be the mean anticipated time to completion. Give a 95% confidence interval for $\mu_1 - \mu_2$.

Statistic	Actual time	Anticipated time	Differences
Sample mean	25.60	13.90	11.70
Sample variance	80.70	52.30	12.80
n	18	18	18

(A) $[10.20, 13.20]$ (B) $[6.37, 17.03]$ (C) $[5.33, 18.07]$
(D)$[9.92, 13.48]$ (E) $[8.84, 14.56]$

[1] 4. The breaking strength of yarn from two different manufacturers is being investigated. A random sample of $n_1 = 30$ observations from the first manufacturer gave a mean of 88 psi and a standard deviation of 5.6 psi. We collected a small sample of size $n_2 = 6$ observations from the second manufacturer. Here are the data :

$$89.8, \quad 90.9, \quad 97.0, \quad 89.2, \quad 95.3, \quad 93.8.$$

Compute the estimated standard error for the estimate of the difference in mean breaking strength.

(A) 28.229 (B) 2.724 (C) 1.650 (D) 5.313 (E) 2.423

[1] 5. A machine is used to fill plastic bottles with dishwashing detergent. The mean and the standard deviation of the fill volume are known : $\mu = 916$ ml and $\sigma = 5$ ml. If we measure the fill volume for $n = 100$ bottles, give the approximate probability that the sample mean fill volume will be between 915.25 ml and 916.75 ml.

(A) 0.0011 (B) 0.0668 (C) 0.8664 (D) 0.9332 (E) 0.1336

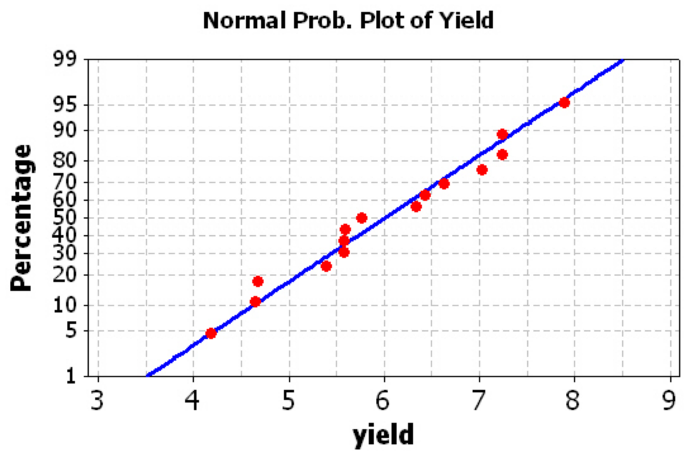
[1] 6. Let A, B and C be mutually independent events such that $P(A \cap B) = P(A \cap C) = P(B \cap C) = 0.3$. Find $P(A \cap B \cap C)$.

(A) 0.027 (B) 0.1643 (C) 0.3 (D) 0.21 (E) 0.9

[1] 7. One process of making green gasoline takes biomass in the form of sucrose and converts it into gasoline using catalytic reactions. At one step in a pilot plant process, the output includes carbon chains of length 3. The descriptive statistics for 15 runs are

n	mean	stdev	Q1	Q2	Q3
15	6.009	1.078	5.390	5.760	7.020

Below is a normal probability plot for the 15 yields. Give a range for the p-value for testing that the population mean is larger than 5.5. At a level of significance of 5%, do we have sufficient evidence that the mean is larger than 5.5 ?



(A) $p < 0.025$; sufficient evidence that $\mu > 5.5$.
(B) $p < 0.025$; insufficient evidence to conclude that $\mu > 5.5$.
(C) $0.025 < p < 0.05$; sufficient evidence that $\mu > 5.5$.
(D) $0.025 < p < 0.05$; insufficient evidence to conclude that $\mu > 5.5$.
(E) $p > 0.05$; insufficient evidence to conclude that $\mu > 5.5$.

[1] 8. Suppose that we wish to determine whether a rare, but very costly, flaw is present. Let us assume that the probability that it is present is 0.01. A fairly simple procedure is proposed to test for this flaw. However, the test is preliminary, as the probabilities of reaching a wrong conclusion are large. When no flaw is present, the probability that the test indicates a flaw is 5%. When a flaw is present, the probability that the test indicates the absence of a flaw is 3%. Given that the test indicates a flaw, find the probability that there really is a flaw.

(A) 0.1639 (B) 0.6250 (C) 0.97 (D) 0.95 (E) 0.8361

[1] 9. Below is the number of MacGuffins purchased over 6 days in some hunting supplies store in Scotland.

5 2 5 3 4 7

Find the first and third quartile for the above data.

(A) Q1=2.167 ; Q3=6.5 (B) Q1=2.75 ; Q3=5.5 (C) Q1=2 ; Q3=4
(D) Q1=2 ; Q3=7 (E) Q1=2.58 ; Q3=6.08

[1] 10. The following data are the temperatures of effluent at discharge from a sewage treatment plant on 15 consecutive days :

41 47 49 48 51 49 43 49
45 51 46 48 51 48 49

We ordered the values from smallest to largest :

41 43 45 46 47 48 48 48
49 49 49 49 51 51 51

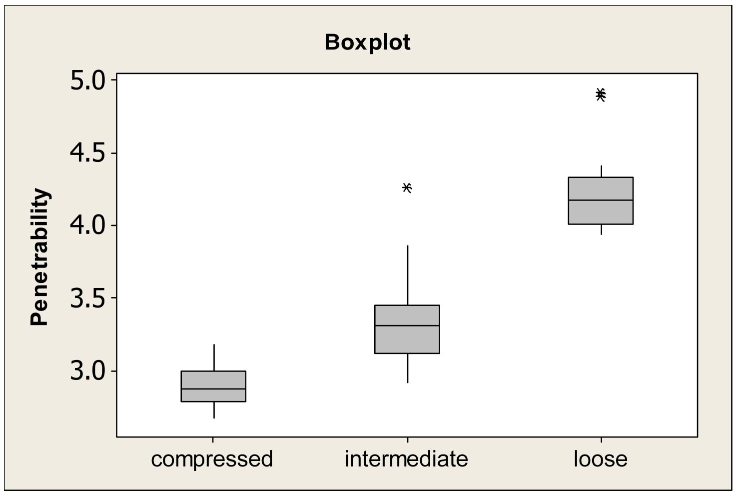
Here are descriptive statistics :

n	mean	StDev	Q1	Q2	Q3
15	47.667	2.895	46.000	48.000	49.000

Are there any outliers in the sample ?

(A) Only 41 is an outlier.
(B) Only 51 is an outlier.
(C) Both 41 and 51 are outliers.
(D) There are no outliers.
(E) None of the preceding.

[**1**] 11. Driving large farm equipment on wet soil compresses the soil. We believe that this will injure future crops. To verify this claim, we measure penetrability on the same soil at three levels of compression. Penetrability is a measure of how much resistance plant root will meet when they try to grow through the soil. We produced the following boxplots.



Choose the true statement. Only one statement is true.

(A) There is an outlier in the random sample of **compressed** soil.

(B) Compression of the soil has no effect on the median of penetrability.

(C) There are no outliers in the random sample of **intermediate** compression.

(D) These graphs are meaningless and should not be used to compare the penetrability for these three types of compressions.

(E) The median of penetrability is largest for the **loose** soil, then for the **intermediate**, and the **compressed** soil has the smallest median penetrability.

[1] 12. A company buys a used piece of equipment whose lifetime is known to follow an exponential distribution with mean of 15 years. Knowing that the piece of equipment is 8 years old at the moment of purchase, what is the probability that it will last another 12 years from that point ?

(A) 0.5866 (B) 0.7659 (C) 0.2636 (D) 0.0367 (E) 0.4493

[1] 13. The labour time required to produce an order of automobile mufflers using a heavy stamping machine is normally distributed with a mean of 4.5 hours and a standard deviation of 0.56 hour. Find a time x (in hours) such that there is a probability of 98% that the time to produce an order is less than x hours.

(A) 5.648 (B) 5.553 (C) 5.454 (D) 3.352 (E) 6.101

[1] 14. A fuel-economy study was conducted for two German automobiles, Mercedes and Volkswagen. One vehicule of each brand is selected. Per tank, suppose that the mean mileage (in miles per gallon) for Mercedes is $\mu_1 = 23$ mpg, while it is $\mu_2 = 38.2$ mpg for Volkswagen. Per tank, the standard deviation of the mileage for Mercedes is 0.9 mpg, while it is 3.3 mpg for Volkswagen.

For each car, the mileage will be observed for 45 tanks. We denote the sample mean mileage (in mpg) for Mercedes as $\overline{X}_1$ and $\overline{X}_2$ will be the mean mileage for Volkswagen. Assuming that the above population parameters are correct, approximate the following probability :

$$P(\overline{X_2} - \overline{X}_1 > 16).$$

(A) 0.9418 (B) 0.0582 (C) 0.9955 (D) 0.0045 (E) 0.9750