



## Statistics for Management - ADM2303

### Midterm Exam - Fall 2014

Time 13:00hrs - Date October 26

Duration: 2 hours

Professors: Davood Astaraky, Jonathan Li, Suren Phansalker, Rui Zhang

**Last Name:** \_\_\_\_\_ **First Name:** \_\_\_\_\_

**Student #:** \_\_\_\_\_ **Section :** \_\_\_\_\_

1. Write in your last and first name, and your Student ID number in the spaces above, and sign the Statement of Academic Integrity below.
2. This exam booklet contains 5 problems. If yours does not, please inform professor now.
3. One page (8 ½ by 11 inches) review sheet, both sides, is permitted.
4. Please note : “Proctors are unable to respond to queries about the interpretation of exam questions. Do your best to answer exam questions as written.”
5. Write the answers in the space provided, continue on the backs of pages if needed.
6. Simple calculators are permitted for arithmetic use only.
7. **NO COMMUNICATION DEVICES** (computers, phones, etc.) **MAY BE WITHIN SIGHT.**
8. **SHOW YOUR WORK CLEARLY.** Correct answers without clear work showing how you got there will not receive full marks.

| Question | Points | Out of |
|----------|--------|--------|
| 1        |        | 9      |
| 2        |        | 16     |
| 3        |        | 25     |
| 4        |        | 25     |
| 5        |        | 25     |
| TOTAL    |        | 100    |

#### Statement of Academic Integrity

The Telfer School of Management does not condone academic fraud, an act by a student that may result in a false academic evaluation of that student or of another student. Without limiting the generality of this definition, academic fraud occurs when a student commits any of the following offences: plagiarism or cheating of any kind, use of books, notes, mathematical tables, dictionaries or other study aid unless an explicit written note to the contrary appears on the exam, to have in his/her possession cameras, radios (radios with head sets), tape recorders, pagers, cell phones, or any other communication device which has not been previously authorized in writing.

#### Statement to be signed by the student:

I have read the text on academic integrity and I pledge not to have committed or attempted to commit academic fraud in this examination.

Signed: \_\_\_\_\_

Note: an examination copy or booklet without that signed statement will not be graded and will receive a midterm exam grade of zero.

**Question 1.** (9 points)

As the election for a city mayor is approaching, many surveys have been conducted recently on citizens' views of what can be improved for the city. The table below summarizes a survey, where a randomly selected citizen was asked for his/her opinion of the necessity of light rail. Citizens are categorized according to the region they are from.

| Views on light rail | Region |       |      |
|---------------------|--------|-------|------|
|                     | West   | South | East |
| Very necessary      | 18%    | 24%   | 20%  |
| Necessary           | 6%     | 12%   | 13%  |
| Not necessary       | 2%     | 2%    | 1%   |
| Not sure            | 1%     | 0%    | 1%   |

Consider the event of “having light rail is necessary” and the event of “citizen coming from the west region”. Are these two events :

- a) Mutually exclusive? State your reason clearly. (3 points)

Answer:

Because  $P(\text{necessary} \ \& \ \text{west})$  is not zero, so they are not mutually exclusive.

- b) Independent? State your reason clearly. (6 points)

Answer:

$$P(\text{necessary} \ \& \ \text{west}) = 6\%$$

$$P(\text{necessary}) = (6\% + 12\% + 13\%)$$

$$P(\text{west}) = (18\% + 6\% + 2\% + 1\%)$$

$$P(\text{necessary}) * P(\text{west}) = (6\% + 12\% + 13\%) * (18\% + 6\% + 2\% + 1\%) = 8.37\%$$

$P(\text{necessary} \ \& \ \text{west})$  and  $P(\text{necessary}) * P(\text{west})$  are not the same, so they are not independent.

As a rule of thumb, if all steps for calculation are correct but the final answers are incorrect numerically, take out at most 1 point.

Students may choose to use  $P(\text{necessary}) \stackrel{?}{=} P(\text{necessary} \mid \text{west region})$  to check independence, or  $P(\text{west}) \stackrel{?}{=} P(\text{west} \mid \text{necessary})$ . In these cases, the calculation of marginal probability, i.e.  $P(\text{necessary})$  (or  $P(\text{west})$ ) counts 2 points, and the calculation of conditional probability 3 points.

**Question 2. (16 points)**

60% of Canadians pass the driving test on their first tries. To make sure they can pass the test, some Canadians would take some driving lessons before taking the test. A survey of those who passed the driving test indicates that 89% of them once took some driving lessons. Another survey of those who did not pass the test indicates that 6% of them once took some driving lessons.

- a) Suppose that one of your friends went to a driving school. What is the probability that (s)he will pass the driving test? (8 points)

Answer:

$$P(\text{pass}) = 60\%.$$

$$P(\text{not pass}) = 40\%$$

$$P(\text{take lesson} | \text{pass}) = 89\%,$$

$$P(\text{take lesson} | \text{did not pass}) = 6\%$$

$$P(\text{pass} | \text{take lesson})$$

$$= P(\text{pass} \& \text{take lesson}) / P(\text{take lesson})$$

$$= P(\text{pass} \& \text{take lesson}) / (P(\text{pass} \& \text{take lesson}) + P(\text{not pass} \& \text{take lesson}))$$

$$= P(\text{pass})P(\text{take lesson} | \text{pass}) / (P(\text{pass})P(\text{take lesson} | \text{pass}) + P(\text{not pass})P(\text{take lesson} | \text{not pass}))$$

$$= 60\% * 89\% / (60\% * 89\% + 40\% * 6\%) = 95.6\%$$

If students skip few steps but provide enough evidence they know how to solve the problem overall, they can still get full mark. For example, students may use a probability tree to solve the problem without explicitly using the formula. The above part marks are given to students who cannot complete their answers.

- b) Suppose that one of your friends decided not to take any driving lesson. What is the probability that (s)he will pass the driving test? (8 points)

Answer:

$$P(\text{pass}) = 60\%.$$

$$P(\text{not pass}) = 40\%$$

$$P(\text{not take lesson} | \text{pass}) = 11\%,$$

$$P(\text{not take lesson} | \text{did not pass}) = 94\%$$

$$P(\text{pass} | \text{not take lesson})$$

$$= P(\text{pass} \& \text{not take lesson}) / P(\text{not take lesson})$$

$$= P(\text{pass} \& \text{not take lesson}) / (P(\text{pass} \& \text{not take lesson}) + P(\text{not pass} \& \text{not take lesson}))$$

$$= P(\text{pass})P(\text{not take lesson} | \text{pass}) / (P(\text{pass})P(\text{not take lesson} | \text{pass}) + P(\text{not pass})P(\text{not take lesson} | \text{not pass}))$$

$$= 60\% * 11\% / (60\% * 11\% + 40\% * 94\%) = 14.93\%$$

If students skip few steps but provide enough evidence they know how to solve the problem overall, they can still get full mark. For example, students may use a probability tree to solve the problem without explicitly using the formula. The above part marks are given to students who cannot complete their answers.

**Question 3.** (25 points)

David's Jeep Tours offer a popular half-day trip in a tourist area. There must be at least 2 passengers for the trip to run, and the vehicle will hold up to 6 passengers. The number of passengers  $X$  on a randomly selected day has the following probability distribution.

|                      |      |      |      |      |      |
|----------------------|------|------|------|------|------|
| Number of Passengers | 2    | 3    | 4    | 5    | 6    |
| Probability          | 0.15 | 0.25 | 0.35 | 0.20 | 0.05 |

- a) Find the mean and standard deviation of random variable  $X$ . (5 Points)

Answer:

$$\mu_X = 2(0.15) + 3(0.25) + 4(0.35) + 5(0.2) + 6(0.05) = 3.75$$

$$\sigma_X = \{0.15(2 - 3.75)^2 + \dots + 0.05(6 - 3.75)^2\}^{\frac{1}{2}} = 1.09$$

- b) David charges \$150 per passenger. Let random variable  $C$  = the total amount of money that David collects on a randomly selected trip. Write the equation that relates  $C$  to the random variable  $X$  and find the mean and standard deviation of random variable  $C$ . (5 Points)

Answer:

$$C = 150X \rightarrow \mu_C = 150\mu_X = 150(3.75) = 562.5\$$$

$$\text{Var}(C) = 150^2 \text{Var}(X) \Rightarrow \sigma_C = 150\sigma_X = 150(1.09) = 163.5\$$$

David's sister Erin, who lives near a tourist area in another part of the country, is impressed by the success of David's business. She decided to join the business, running tours on the same days as David in her slightly smaller vehicle, under the name Erin's adventures. After a year of steady booking, Erin discovers that the number of passengers  $Y$  on her half-day tours has the mean  $\mu_Y = 3.1$  and standard deviation.  $\sigma_Y = 0.943$ .

- c) Let random variable  $T$  = the total passengers that David and Erin have on a given day. How does this compare to the random variable  $X$  and  $Y$ ? Find the mean and standard deviation of random variable  $T$ . (7.5 points)

Answer:

$$T = X + Y \rightarrow \mu_T = \mu_X + \mu_Y = 3.75 + 3.1 = 6.85$$

$$\sigma_T = \sqrt{\sigma_X^2 + \sigma_Y^2} = \sqrt{1.09^2 + 0.943^2} = 1.441$$

- d) Erin charges \$175 per passenger for her trip. Let random variable  $G$  = the total amount that David and Erin collect on a randomly chosen day. How does this relate to the random variable  $X$  and  $Y$ ? Find the mean and standard deviation for random variable  $G$ . (7.5 points)

Answer:

$$G = 150X + 175Y \rightarrow \mu_G = 150\mu_X + 175\mu_Y = 150(3.75) + 175(3.1) = 1105\$$$

$$\sigma_T = \sqrt{150^2 \times \sigma_X^2 + 175^2 \times \sigma_Y^2} = \sqrt{150^2 \times 1.09^2 + 175^2 \times 0.943^2} = 232.3\$$$

**Question 4.** (25 points)

A community restaurant offers a variety of fast foods during lunch times. The restaurant offers meat-only menus and vegetable-only menus.

- a) Suppose that 90% of customers would prefer to have meat-only menus. What is the probability that the restaurant will receive three orders on vegetable-only menus from five customers who just walked in? (6 points)

Answer:

Probability of a customer ordering vegetable-only menu

is  $p = (1-90\%) = 0.1$

Apply **Binomial distribution** with  $n = 5$  and  $p = 0.1$ .

Detailed calculation of  $P(X=3) = 5!/[3!(5-3)!]*0.1^3*(1-0.1)^{(5-3)} = 0.0081$ .

On a general weekday, there are in average 6 customers during the peak hours (11:00am – 1:00pm).

- b) What is the probability that the restaurant has more than two customers during the peak hours (i.e., 11:00am – 1:00pm)? (7 points)

Answer:

Apply Poisson model  
with average arrival rate of 6.

$$P(\text{more than two}) \\ = 1 - P(\text{no more than 2 customers})$$

$$P(\text{no more than 2 customers}) \\ = P(0) + P(1) + P(2) \\ = e^{(-6)} * 6^{(0)} / (0!) + e^{(-6)} * 6^{(1)} / (1!) + e^{(-6)} * 6^{(2)} / (2!) = 0.062$$

The answer is 0.938

- c) Suppose that the restaurant has received four lunch orders by 12:00pm. What is the probability that the restaurant will receive another four customers in the rest of peak hours (i.e., 12:00pm – 1:00pm)? (6 points)

Answer:

Apply Poisson model  
with average arrival rate of  $6/2 = 3$   
The probability for another four customers is  
(given that 4 orders have been received)

$$P(4) \\ = e^{(-3)} * 3^{(4)} / (4!) = 0.168$$

- d) Suppose that half of the customers are male. What is the probability that there is no female customer in the first half hour of peak hours, (i.e., 11:00am – 11:30am)? (6 points)

Answer:

Apply Poisson model  
with average arrival rate of  $6/4/2 = 0.75$ .  
The probability that no female customer arrives  
is  $e^{(-0.75)} * 0.75^{(0)} / (0!) = e^{(-0.75)} = 0.4723$ .

### Question 5 (25 points)

Use the following information for sub-questions 'a', 'b' and 'c'. Here B\$ indicates billions of dollars.

In a lecture class for 'International Trade', the professor at a University stated that the Exports and Imports during one year of a given country are estimated to have mean values of 80 B\$ and 100 B\$ with standard deviations of 15B\$ and 20 B\$ respectively. He also stated that the exports as well as imports can both be considered to follow normal distributions.

- a) Calculate the probability that the imports would be more than 110 B\$ during the given year. (6 points)
- b) At the end of the lecture, he mumbled that there is only a 10% probability that the exports would be less than some figure that you could not hear. Calculate that value he mentioned/mumbled? (6 points)
- c) Trade Surplus is defined as Exports minus Imports. Calculate the probability that there will be a positive trade surplus. (8 points)
- d) The estimates for the standard deviations for both Exports and Imports were slightly modified downwards and were 12 B\$ and 16 B\$ respectively. The mean values were unchanged. Taking these changes into account, re-calculate the probability that there will be a positive trade surplus. (5 points)



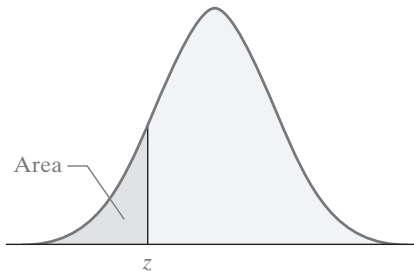


Table IV

| Standard Normal Distribution |        |        |        |        |        |        |        |        |        |        |
|------------------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $z$                          | .00    | .01    | .02    | .03    | .04    | .05    | .06    | .07    | .08    | .09    |
| -3.4                         | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0003 | 0.0002 |
| -3.3                         | 0.0005 | 0.0005 | 0.0005 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0004 | 0.0003 |
| -3.2                         | 0.0007 | 0.0007 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0006 | 0.0005 | 0.0005 | 0.0005 |
| -3.1                         | 0.0010 | 0.0009 | 0.0009 | 0.0009 | 0.0008 | 0.0008 | 0.0008 | 0.0008 | 0.0007 | 0.0007 |
| -3.0                         | 0.0013 | 0.0013 | 0.0013 | 0.0012 | 0.0012 | 0.0011 | 0.0011 | 0.0011 | 0.0010 | 0.0010 |
| -2.9                         | 0.0019 | 0.0018 | 0.0018 | 0.0017 | 0.0016 | 0.0016 | 0.0015 | 0.0015 | 0.0014 | 0.0014 |
| -2.8                         | 0.0026 | 0.0025 | 0.0024 | 0.0023 | 0.0023 | 0.0022 | 0.0021 | 0.0021 | 0.0020 | 0.0019 |
| -2.7                         | 0.0035 | 0.0034 | 0.0033 | 0.0032 | 0.0031 | 0.0030 | 0.0029 | 0.0028 | 0.0027 | 0.0026 |
| -2.6                         | 0.0047 | 0.0045 | 0.0044 | 0.0043 | 0.0041 | 0.0040 | 0.0039 | 0.0038 | 0.0037 | 0.0036 |
| -2.5                         | 0.0062 | 0.0060 | 0.0059 | 0.0057 | 0.0055 | 0.0054 | 0.0052 | 0.0051 | 0.0049 | 0.0048 |
| -2.4                         | 0.0082 | 0.0080 | 0.0078 | 0.0075 | 0.0073 | 0.0071 | 0.0069 | 0.0068 | 0.0066 | 0.0064 |
| -2.3                         | 0.0107 | 0.0104 | 0.0102 | 0.0099 | 0.0096 | 0.0094 | 0.0091 | 0.0089 | 0.0087 | 0.0084 |
| -2.2                         | 0.0139 | 0.0136 | 0.0132 | 0.0129 | 0.0125 | 0.0122 | 0.0119 | 0.0116 | 0.0113 | 0.0110 |
| -2.1                         | 0.0179 | 0.0174 | 0.0170 | 0.0166 | 0.0162 | 0.0158 | 0.0154 | 0.0150 | 0.0146 | 0.0143 |
| -2.0                         | 0.0228 | 0.0222 | 0.0217 | 0.0212 | 0.0207 | 0.0202 | 0.0197 | 0.0192 | 0.0188 | 0.0183 |
| -1.9                         | 0.0287 | 0.0281 | 0.0274 | 0.0268 | 0.0262 | 0.0256 | 0.0250 | 0.0244 | 0.0239 | 0.0233 |
| -1.8                         | 0.0359 | 0.0351 | 0.0344 | 0.0336 | 0.0329 | 0.0322 | 0.0314 | 0.0307 | 0.0301 | 0.0294 |
| -1.7                         | 0.0446 | 0.0436 | 0.0427 | 0.0418 | 0.0409 | 0.0401 | 0.0392 | 0.0384 | 0.0375 | 0.0367 |
| -1.6                         | 0.0548 | 0.0537 | 0.0526 | 0.0516 | 0.0505 | 0.0495 | 0.0485 | 0.0475 | 0.0465 | 0.0455 |
| -1.5                         | 0.0668 | 0.0655 | 0.0643 | 0.0630 | 0.0618 | 0.0606 | 0.0594 | 0.0582 | 0.0571 | 0.0559 |
| -1.4                         | 0.0808 | 0.0793 | 0.0778 | 0.0764 | 0.0749 | 0.0735 | 0.0721 | 0.0708 | 0.0694 | 0.0681 |
| -1.3                         | 0.0968 | 0.0951 | 0.0934 | 0.0918 | 0.0901 | 0.0885 | 0.0869 | 0.0853 | 0.0838 | 0.0823 |
| -1.2                         | 0.1151 | 0.1131 | 0.1112 | 0.1093 | 0.1075 | 0.1056 | 0.1038 | 0.1020 | 0.1003 | 0.0985 |
| -1.1                         | 0.1357 | 0.1335 | 0.1314 | 0.1292 | 0.1271 | 0.1251 | 0.1230 | 0.1210 | 0.1190 | 0.1170 |
| -1.0                         | 0.1587 | 0.1562 | 0.1539 | 0.1515 | 0.1492 | 0.1469 | 0.1446 | 0.1423 | 0.1401 | 0.1379 |
| -0.9                         | 0.1841 | 0.1814 | 0.1788 | 0.1762 | 0.1736 | 0.1711 | 0.1685 | 0.1660 | 0.1635 | 0.1611 |
| -0.8                         | 0.2119 | 0.2090 | 0.2061 | 0.2033 | 0.2005 | 0.1977 | 0.1949 | 0.1922 | 0.1894 | 0.1867 |
| -0.7                         | 0.2420 | 0.2389 | 0.2358 | 0.2327 | 0.2296 | 0.2266 | 0.2236 | 0.2206 | 0.2177 | 0.2148 |
| -0.6                         | 0.2743 | 0.2709 | 0.2676 | 0.2643 | 0.2611 | 0.2578 | 0.2546 | 0.2514 | 0.2483 | 0.2451 |
| -0.5                         | 0.3085 | 0.3050 | 0.3015 | 0.2981 | 0.2946 | 0.2912 | 0.2877 | 0.2843 | 0.2810 | 0.2776 |
| -0.4                         | 0.3446 | 0.3409 | 0.3372 | 0.3336 | 0.3300 | 0.3264 | 0.3228 | 0.3192 | 0.3156 | 0.3121 |
| -0.3                         | 0.3821 | 0.3783 | 0.3745 | 0.3707 | 0.3669 | 0.3632 | 0.3594 | 0.3557 | 0.3520 | 0.3483 |
| -0.2                         | 0.4207 | 0.4168 | 0.4129 | 0.4090 | 0.4052 | 0.4013 | 0.3974 | 0.3936 | 0.3897 | 0.3859 |
| -0.1                         | 0.4602 | 0.4562 | 0.4522 | 0.4483 | 0.4443 | 0.4404 | 0.4364 | 0.4325 | 0.4286 | 0.4247 |
| -0.0                         | 0.5000 | 0.4960 | 0.4920 | 0.4880 | 0.4840 | 0.4801 | 0.4761 | 0.4721 | 0.4681 | 0.4641 |