

**CONCORDIA UNIVERSITY**  
**Department of Mathematics & Statistics**

Course	Number	Section(s)
Mathematics	209	All except EC
Examination	Date	Pages
Final	December 2013	3
Instructors	Course Examiner	
E. Duma, E. Lee, M. Padamadan	R. Raphael	
R. Raphael, R. Rodriguez, F. Romanelli, C. Santana		
Special Instructions		
▷ Ruled booklets to be used.		
▷ Only approved calculators allowed.		

[MARKS]

[6] 1. Find the following limits:

$$(a) \lim_{x \rightarrow -2} \frac{2x^2 + 9x + 10}{x + 2} \quad (b) \lim_{x \rightarrow 0} \frac{\sqrt{49 + x} - 7}{x} \quad (c) \lim_{x \rightarrow \infty} \frac{2x^3 + 3x^2 - 9}{-\frac{1}{3}x^3 + 5x + 7}$$

[18] 2. Find the derivative for each of the following (do not simplify):

(a)  $y = 8x^3 + 7x^6 + 12$

(b)  $y = \frac{2}{3}x^{-5} - 5\sqrt{x} + 5$

(c)  $y = (2x^2 + 3)(x^3 + x - 2)^3$

(d)  $y = \frac{-2x^3 + x^2}{x^2 + x + 1}$

(e)  $y = e^{\ln(7x)}$

(f)  $y = \left( e^{3x^2 + 15x} \right) (\ln(7x^3 - 4x))$

(g)  $y = e^{3x^2} \ln(x^3 + 6)^2$

(h) Find  $y'$  and evaluate at  $(1, 1)$ :  $2y + x \ln(y) = 2x^3$

- [10] 3. A company manufactures automatic transmissions for automobiles. The total weekly cost (in dollars) of producing  $x$  transmissions is given by

$$C(x) = 50,000 + 600x - 0.75x^2$$

- (a) Find the marginal cost function.
- (b) Find the marginal cost at a production level of 200 transmissions per week and interpret the results.
- (c) Find the exact cost at producing the 201st transmission.

- [18] 4. For the function  $f(x) = x^4 + 4x^3$  find:

*[Please list the following neatly]*

- (a) the intervals where  $f(x)$  is increasing;
- (b) the intervals where  $f(x)$  is decreasing;
- (c) the intervals where  $f(x)$  is concave up;
- (d) the intervals where  $f(x)$  is concave down;
- (e) the local maximum;
- (f) the local minimum;
- (g) the inflection point(s);
- (h)  $\lim_{x \rightarrow +\infty} f(x)$ ;
- (i)  $\lim_{x \rightarrow -\infty} f(x)$ ;
- (j) Using the above results, sketch the graph of  $f(x)$ .
- [6] 5. Find the absolute extrema of  $f(x) = x^4 - 4x^3 + 5$  on the interval  $[0, 4]$ .
- [6] 6. Find the equation of the line tangent to the  $y = -3e^{x^2} + 5$  where  $x = 0$ .

[6] 7. Evaluate the following; answers must be accurate to 3 decimals:

(a)  $\int_0^3 4x^2 dx$

(b)  $\int_1^2 (2x + 3e^x - \frac{4}{x}) dx$

(c)  $\int_0^1 xe^{-x^2} dx$

[10] 8. Compute the antiderivatives:

(a)  $\int (4t^4 - t^3 + 5t) dt$

(b)  $\int (-\frac{3}{x} - x^{-12}) dx$

(c)  $\int xe^{-x^2} dx$

(d)  $\int (x^3 + x)e^{(x^4+2x^2)} dx$

(e)  $\int (x^2 - 2)(x + 3) dx$

[10] 9. Find the area bounded by  $y = x^3$  and  $y = 4x$ .

[10]10. The Gini index of a country is  $\frac{1}{6}$ . Its Lorenz curve has the form  $f(x) = ax + \frac{1}{2}x^2$ . Find  $a$ .

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