MATH 1107 A - Summer 2015 Final Practice Problems

1. In this question, $z = 1 + i$, $w = 2 + i$. Give the answer to each part. Simplify your answer as much as possible. All complex numbers must be in **rectangular/standard form**.

   (a) $\overline{w} =$

   (b) $\mathbf{Im}(w) =$

   (c) $z - w =$

   (d) $w^{-1} =$

   (e) $|w| =$

   (f) One of the cube roots of z, with real and imaginary parts rounded to 4 decimal places, is $-0.2905 + bi$. Then $b =$

   (g) Let $A = \begin{bmatrix} 1 & 1 \\ 1 & i \end{bmatrix}$. Then $\det(A^{10}) =$

   (h) Let $B = \begin{bmatrix} 2 & -2 & 0 \\ 2 & 0 & -2 \\ 0 & 1 & -1 \end{bmatrix}$. The rank of B is

   (i) Let $A = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$, and $C = A^{\mathsf{T}} + 2B^{-1}$.
   Then the entry $C_{1,2} =$

   (j) The eigenvalues of $\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$ are

   (k) Let W be a proper subspace of the vector space of 2×3 matrices with real entries.

   The largest possible value for the dimension of W is

   (l) Let $\Gamma = \left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$ be an ordered basis for $\mathbb{R}^2$. Let $u = \begin{bmatrix} a \\ b \end{bmatrix}$ be a vector in $\mathbb{R}^2$.
   Suppose that $[u]_\Gamma = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Then $a + b =$

   (m) Let T be a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}$ such that $T\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right) = 1$ and $T\left(\begin{bmatrix} 3 \\ -1 \end{bmatrix}\right) =$
   -1. Then $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) =$

2. Let $A \in \mathbb{R}^{n\times n}$ be such that $\det(A) \neq 0$. For what positive integers n is $\det(-A) = \det(A)$? Justify your answer.

3. Find all real values a such that the system of linear equations

$$\begin{array}{rcrcrcr} x & - & y & + & z & = & 1 \\ x & - & 2y & + & 2z & = & -1 \\ & & y & - & z & = & a \end{array}$$

has no solutions.

4. Let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ be matrices defined over the real numbers.

   (a) Find a basis for the nullspace of A.

   (b) Find a basis for the column space of A.

   (c) Determine if the nullspace of A is the same as the column space of B. Justify your answer.

5. Consider the matrix

$$A = \begin{bmatrix} k & 1 & 1 & 1 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & k & 1 \\ 1 & -1 & 2 & -1 \end{bmatrix}.$$

Determine all values of k such that A is singular.

6. Consider the linear transformation T from $\mathbb{R}^4$ to the vector space of 2×2 real matrices given by

$$T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} 2a + c & a - d \\ -2a + 2d & b - d \end{bmatrix}$$

   (a) Determine a basis for the range of T.

   (b) Is T surjective? injective? bijective? Justify your answer.

7. Consider the matrix $A = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.

   (a) Determine A^{-1}.

   (b) Diagonalize A by finding a matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

   (c) Use Cramer's Rule to solve the system $Ax = b$ where $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ is a tuple of variables.