Concordia University
Engineering and Computer Science

ENGR 242/4 Statics Winter 2012

Final Exam Solutions

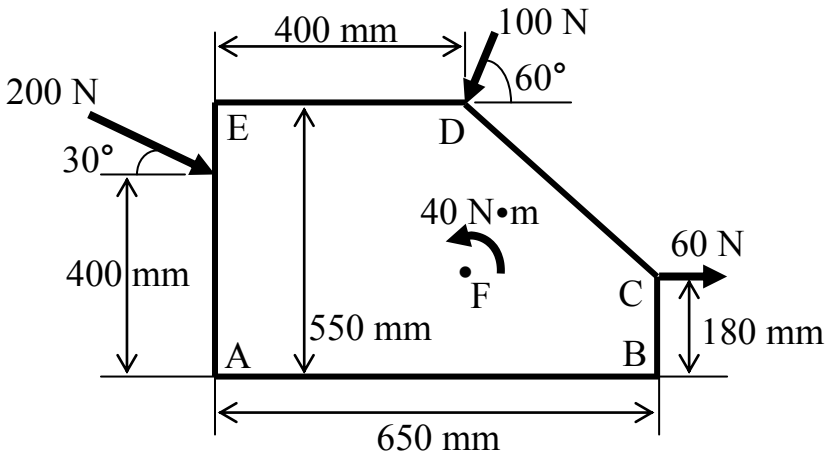
Name (print) _____

Student ID _____

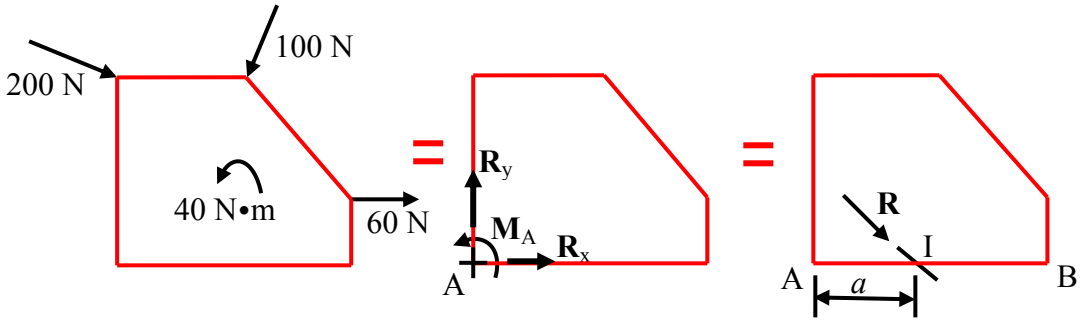
Signature _____

1. A plate is subjected to the forces and the moment as shown.

   (a) Find an equivalent force-couple system at A
   (b) Determine the location of the resultant force by specifying the distance from A of the point of its intersection with line AB.



(Sol.)



(a) Replace the applied forces and moment with an equivalent force-couple system at A

$\sum F_x$: $\quad 200 \cos 30° - 100 \cos 60° + 60 = R_x$

$$R_x = 183.2 \text{ N}$$

$\sum F_y$: $\quad -200 \sin 30° - 100 \sin 60° = R_y$

$$R_y = -186.6 \text{ N}$$

$\sum M_A$: $\quad -(0.40 \text{ m})(200 \text{ N}) \cos 30° - (0.40 \text{ m})(100 \text{ N}) \sin 60°$

$$+ (0.55 \text{ m})(100 \text{ N}) \cos 60° - (0.18 \text{ m})(60 \text{ N}) + 40 \text{ N·m} = M_A$$

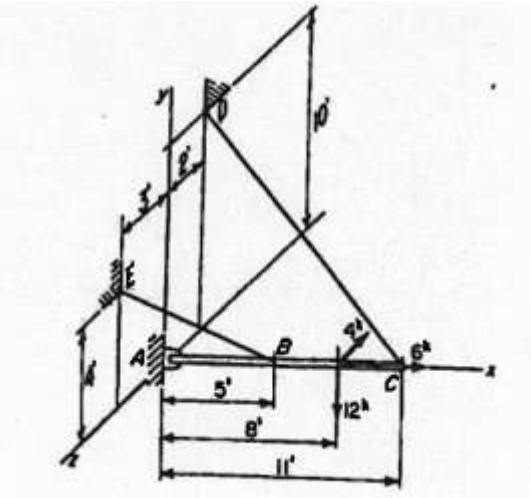
$$M_A = -47.22 \text{ N·m}$$

$$\mathbf{R} = 183.2\,\mathbf{i} - 186.6\,\mathbf{j} \quad , \quad \mathbf{M}_A = -47.22\,\mathbf{k}$$

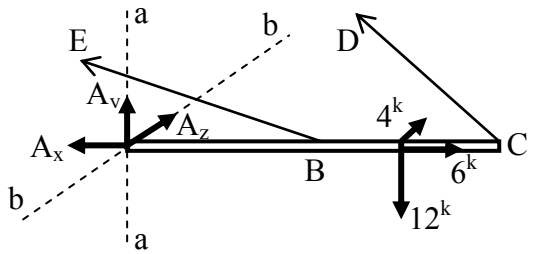
(b) Now with **R** at *I* $\quad \sum M_A : -47.22 \text{ N·m} = -a\,(186.6 \text{ N})$

$$\underline{a = 0.253 \text{ m}}$$

2. The boom AC is supported by a ball-and-socket joint at A and by the cables BE and CD, as shown. Determine the tension in the cable DC.



(**Sol.**)



DC = 15^{FT}

EB = 7.07^{FT}

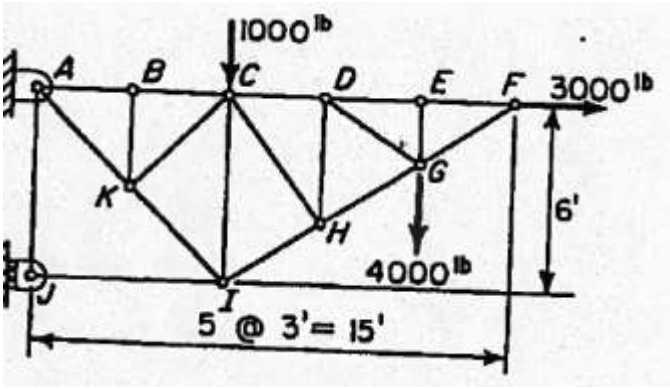
$\sum M_{aa} = 4\,(8) + 11\,(2 / 15)\,D - 5\,(3 / 7.07)\,E = 0$

$\sum M_{bb} = 12\,(8) - 11\,(10 / 15)\,D - 5\,(4 / 7.07)\,E = 0$

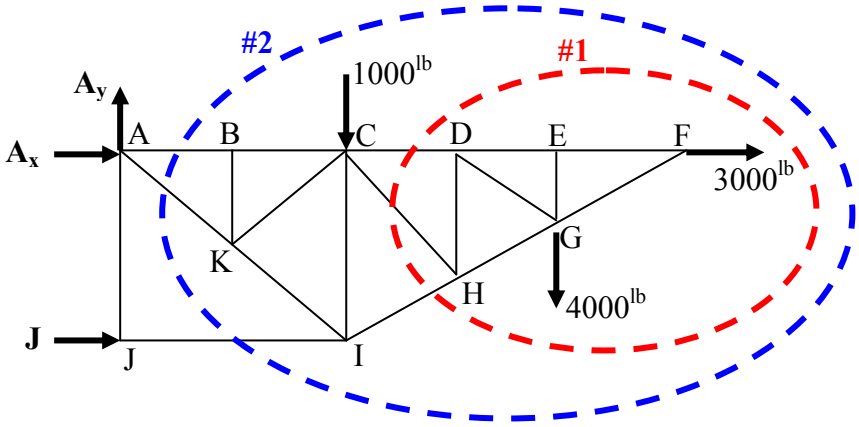
Solving simultaneously for D

$\underline{\mathbf{D} = 5.73^{k}}$

3. Determine the axial forces in members AK, CH and EG of the truss and loading shown. Indicate whether these members are in tension or compression.



(Sol.)



Joint @ E : <u>EG = 0</u>

∵ The forces in two opposite members are equal. The third member force is equal to the load (including zero load).

FBD #1 (Method of Section) : $\sum M_F = 0 = (3/5)\ CH\ (4) + (4/5)\ CH\ (6) - 4000\ (3)$

<u>CH = 1667 lb (Tension)</u>

Complete FBD : $\sum M_A = 0 = 1000\ (6) + 4000\ (12) - 6\ J$

$J = 9000$ lb

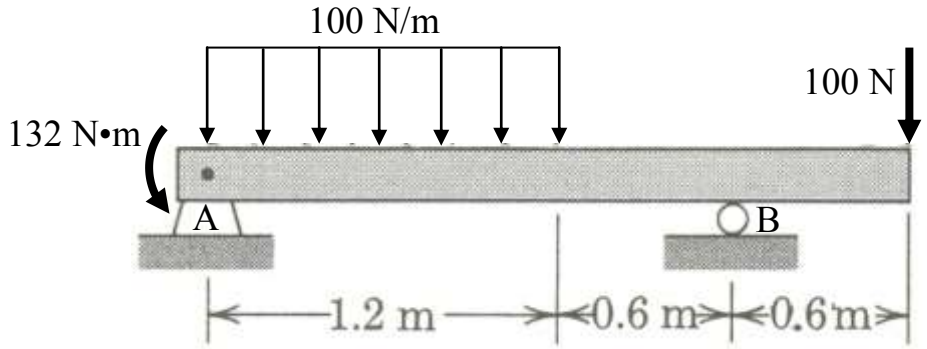
Joint @ J : AJ = 0, JI = – 9000 lb

FBD #2 : $\sum M_F = 0 = 1000\ (9) + 4000\ (3) - (-9000)\ (6)$

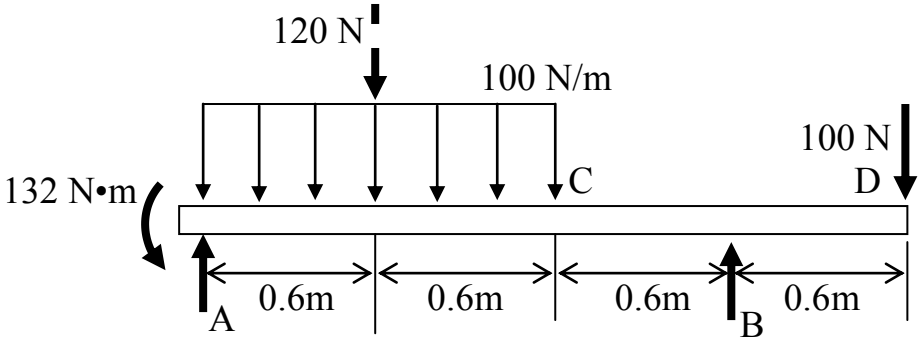
$- (1/\sqrt{2})\ AK\ (12) - (1/\sqrt{2})\ AK\ (3)$

<u>AK = 7070 lb (Tension)</u>

4. Draw the shear force and bending moment diagrams for the simply-supported beam with a cantilever shown. Indicate maximum values.



(Sol.)

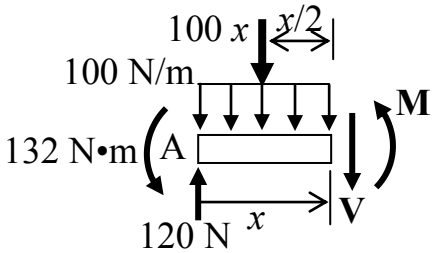


$\sum M_A = 0$: 132Nm – (120N)(0.6m) + B(1.8m) – (100N)(2.4m) = 0

$\therefore$ B=100 N ↑

$\sum F_y = 0$: 100 – 120 + A – 100 = 0 $\therefore$ A=120 N ↑

From A to C:

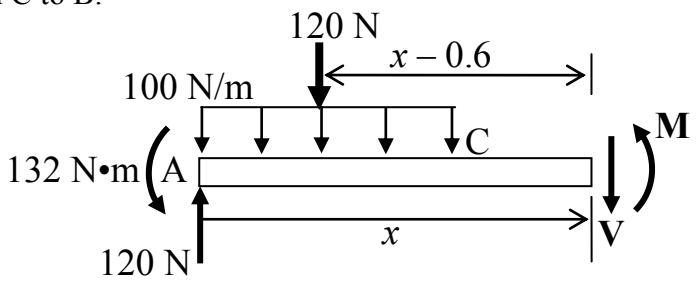


$\sum F_y = 0$: 120 – 100x – V = 0 $\therefore$ V = 120 – 100x

$\sum M_1 = 0$: 132 – 120x + (100x)(x/2) + M = 0

$\therefore$ M = – 50x^2 + 120x – 132

From C to B:

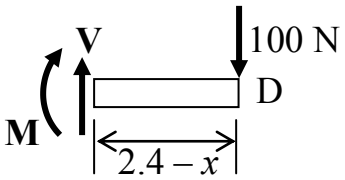


$\sum F_y = 0$: 120 – 120 – V = 0 ∴ V = 0 N

$\sum M_2 = 0$: 132 – 120x + 120(x – 0.6) + M = 0

∴ M = – 60

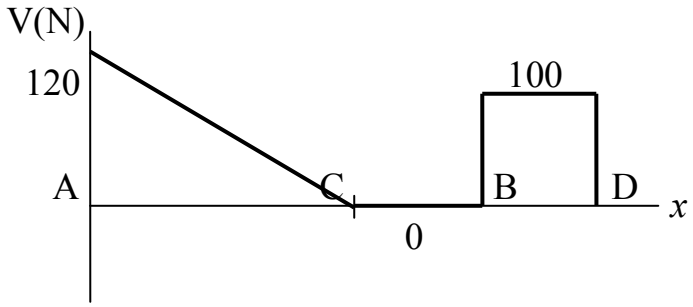
From B to D:

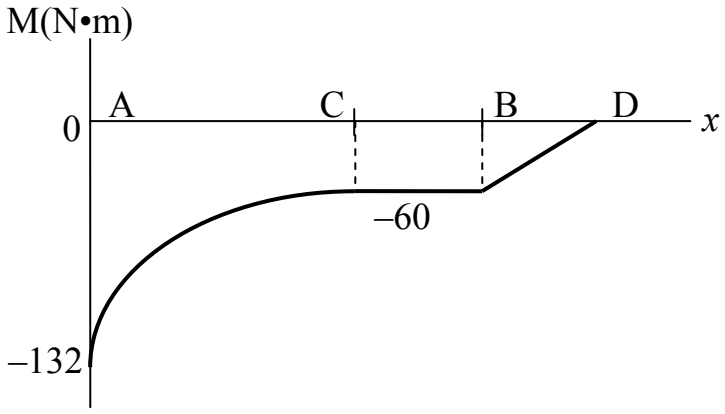


$\sum F_y = 0$: –100 + V = 0 ∴ V = 100 N

$\sum M_3 = 0$: – 100(2.4 – x) – M = 0

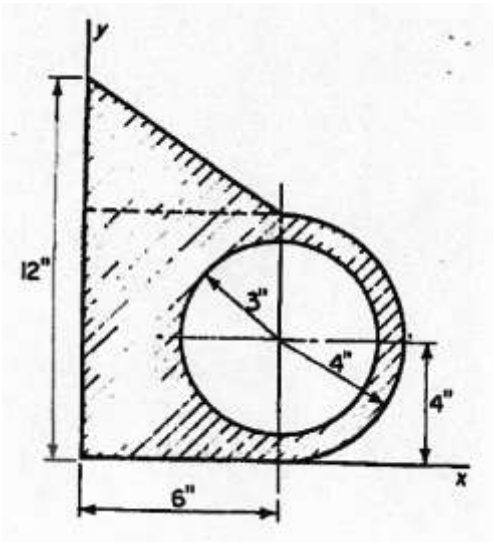
∴ M = 100x – 240



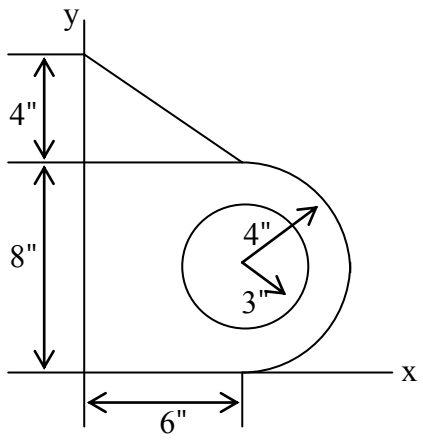


Maximum shear is 120 N and maximum bending moment is 132 N·m

5. For the shaded area shown, determine the radius of gyration with respect to the x-axis.



(**Sol.**)



Area = $12 + 48 + \pi(4)^2/2 - \pi(3)^2 = 60 - \pi = 56.86\ \text{in}^2$

$I_x = (1/3)\,(6)(8)^3 + (1/36)\,(6)(4)^3 + 12\,(8+4/3)^2$
$+ \pi(4)^4/8 + \pi(4)^2\,(4)^2/2 - [\pi(3)^4/4 + \pi(3)^2\,(4)^2] = 2066\ \text{in}^4$

$K_x = \sqrt{\dfrac{2066}{56.86}} = \underline{6.03\ \text{in}}$