CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	**Number**	**Sections**
Mathematics	205	All
Examination	**Date**	**Pages**
Final	April 2014	2
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Special Instructions:	Only approved calculators are allowed **Show all your work for full marks**	

MARKS

[10] **1.** **(a)** Sketch the graph of $f(x) = |x-3|$, and approximate the area between the graph $y = f(x)$ and the x-axis on the interval $[-2, 4]$ by the midpoint Riemann sum using partitioning of the interval into three subintervals of equal length. Compare this approximation with the exact value of $\int_{-2}^{4} f(x)\, dx$.

(b) Use the Fundamental Theorem of Calculus to calculate the derivative of $F(x) = \int_{-x}^{x} e^{-t^3}\, \mathrm{d}t$,

and determine whether $F(x)$ is increasing or decreasing at $x = -2$

[10] **2.** Calculate the following indefinite integrals:

(**a**) $\displaystyle\int \frac{x^2-3}{x^2-9}\, \mathrm{d}x$ (**b**) $\displaystyle\int \sqrt{x}\, \ln x\, \mathrm{d}x$

[12] **3.** Find the antiderivative $F(t)$ of the function $f(t)$ passing through the given point:

(**a**) $f(t) = \left(t - \frac{1}{t}\right)^2$, $F(1) = -1$.(**b**) $f(t) = \frac{e^t}{1+e^{2t}}$, $F(0) = -\frac{\pi}{4}$.

[12] **4.** Evaluate the following definite integrals (give the exact answers):

(**a**) $\displaystyle\int_0^{\pi} \cos^2(x)\, \sin^3(x)\, \mathrm{d}x$ (**b**) $\displaystyle\int_0^{\pi/4} \sqrt{1+8\tan(x)}\, \sec^2(x)\, \mathrm{d}x$

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[5] **5.** Evaluate the improper integral $\int_0^4 \frac{1}{x-2}\,\mathrm{d}x$ or show that it diverges.

[18] **6.** **(a)** Sketch the curves $x = y^2 - 4y$ and $x = 2y - y^2$, find their points of intersection, and find the area enclosed by the curves.

(b) Sketch the region between the curves $y = \cos x$ and $y = \sin x$; $0 \le x \le \pi/4$, and find the volume of the solid generated by rotating this region about the x-axis.

(c) Find the average value of the function $f(x) = \dfrac{x}{\sqrt{9 + x^2}}$ on the interval $[0, 4]$.

[8] **7.** Find the limit of the sequence $\{a_n\}$ or prove that the limit does not exist:

(**a**) $a_n = \dfrac{3n^2\cos(\pi n)}{\sqrt{1+4n^4}}$ (**b**) $a_n = \ln(n+1) - \ln(n)$

[12] **8.** Determine whether the series is divergent or convergent, and if convergent, whether absolutely or conditionally :

(**a**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n}{1+n^2}$ (**b**) $\displaystyle\sum_{n=1}^{\infty} \frac{(-3)^{3n}}{n!}$ (**c**) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^2}$

[5] **9.** Find the interval of convergence of the series

$$\sum_{n=1}^{\infty} \frac{(4x-2)^n}{n+1}$$

[8] **10.** Find **(a)** the radius of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{x^{2n+1}}{8^n}$$

(b) within this radius, the sum of the series as a function of x.

[5] **Bonus Question.** Let $f(x) = \sqrt{6x - x^2}$.

(**a**) Determine the domain $[a, b]$ of f and graph this function.

(**b**) Calculate the definite integral of f over its domain: $\int_a^b f(x)\,\mathrm{d}x$.