CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	ALL
Examination	**Date**	**Pages**
Final	June 2013	2
Instructors		**Course Examiner**
L. Dube, R. Mearns, R. Raphael		R. Raphael

Special Instructions

▷ Only approved calculators are allowed.

MARKS

[12] **1.** (a) Find $\lim_{x \to -3} \dfrac{x^2 + 15x + 36}{x^2 + 10x + 21}$

(b) Find $\lim_{x \to \infty} \dfrac{3x^3 - 2x + 7}{4x^2 + x - 5}$

(c) Give an example of a function f defined for all real numbers which has the property that $\lim_{x \to +\infty} f$ and $\lim_{x \to -\infty} f$ are both equal to $-\infty$.

[8] **2.** Find dh if $h = x^{2.1} - 7$, $x = 3$, and the change in the x is 0.1. What is the value of Δh for these values?

[12] **3.** Find the derivatives of the following functions. YOU DO NOT HAVE TO SIMPLIFY.

(A) $f(x) = e^x - 25x^4 + \frac{2^3}{\sqrt{x^7}}$.

(B) $g(x) = (3\sqrt{x} - \frac{7}{x} + 6x^2 + 2)(5x^3 - ln(x) - e^2\pi)$.

(C) $h(x) = e^{(-2x^3-7)} - \frac{6}{7x} - x^3\ ln(x)$.

(D) $m(x) = [x^4 - 4\ ln(x^2 + 5)]^7$.

[8] **4.** Use implicit differentiation to find $y' = dy/dx$

$$xe^y - y^2\, ln(x) = 6y$$

[10] **5.** A weather balloon is rising vertically at the rate of 5 meters per second. An observer is standing on the ground 300 meters from where the balloon was released. At what rate is the distance between the observer and the balloon changing when the balloon is 400 meters high?

[10] **6.** Use the price-demand equation $0.02x + p = 60$ to find the values of p for which the demand is elastic and for which the demand is inelastic.

[8] **7.** Find the absolute maximum and absolute minimum for the function

$f(x) = x^3 - 12x$ on the interval $[-3,.3]$.

[12] **8.** Graph the sales function $N(x) = -0.25x^4 + 23x^3 - 540x^2 + 80,000$ over the interval $24 \leq x \leq 45$. Determine when N is increasing, when it is decreasing. Does N have a maximum? If so, find it. Does N have a point of inflection? If so, find it.

[10] **9.** Compute the following:

(A) $\int_3^4 (3x - \frac{4}{x} + 2e^x)\, dx.$

(B) $\int e^{-5x}\, dx\,.$

(C) $\int (\frac{x^3}{(x^4+6)^6})\, dx.$

(D) $\int (4x^3 - 7x^6)\, dx.$

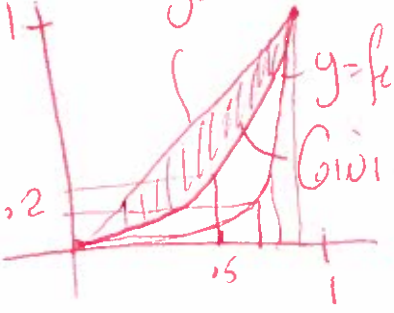
measure of distribution of wealth

[10] **10.** Suppose that a country has Lorenz curve $f(x) = \frac{3}{10}x + \frac{7x^2}{10}$. Find the Gini index.

$(.2, .7)$ 20%, 70%

x - % of pop ; y = % of wealth

$y = x$



$(0.7, 0.3) \Rightarrow$ 70% of pop owns 30% of total wealth

US = 0,4

Sweden = 0,25

Bolivia = 0,61

CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

MATH 209/4 all sections except EC: - Fundamental Mathematics II
Midterm - Saturday, March 5, 2016 (1h30min)

Only approved calculators are permitted.
Justify all your answers.

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1. (a) [7 *marks*] Find

$$\lim_{x\to-\infty} \frac{4x^2-5}{-3x^3+7x-2}$$

   (b) [8 *marks*] Give an example of a function $a(x)$ and a function $b(x)$ with the following properties:

$$(i)\ \lim_{x\to 3} a(x) = 0 \qquad (ii)\ \lim_{x\to 3} b(x) = 0 \qquad (iii)\ \lim_{x\to 3} \frac{[b(x)]^2}{a(x)} = -3$$

2. [7 *marks*] Let $h(x) = -x^3 + x$. Work out the following in detail:

$$\lim_{s\to 0} \frac{h(x+s)-h(x)}{s}$$

3. [12 *marks*]

   (a) If $f(x) = 7x^{4/3} - x^{-8} + 2$, find $f'(x)$. You don't have to simplify the answer.

   (b) If $g(x) = [4x^3 - 7][3 - \ln(x^3)]$, find $g'(x)$. You don't have to simplify the answer.

   (c) Find $h'(x)$ if $h(x) = \dfrac{x^3 - \frac{1}{x}}{x^3 - e^x}$. You don't have to simplify the answer.

   (d) Find the value of dy if $y = \ln(x+2)$, $x = 3$, and the change in x is 0.2.

4. [6 *marks*] A population grew from 12 million to 34 million in 50 years. Assuming continuous compounding, what is the associated annual rate of growth?

5. [10 *marks*] A small machine shop manufactures drill bits used in the petroleum industry. The manager estimates that the total daily cost (in dollars) of producing x bits is

$$C(x) = 1,000 + 25x - 0.1x^2$$

   (a) Find $\overline{C}(x)$ and $\overline{C}'(x)$.

   (b) Find $\overline{C}(10)$ and $\overline{C}'(10)$. Interpret these quantities.

   (c) Use the results in part (b) to estimate the average cost per bit at a production level of 11 bits per day.

PLEASE TURN OVER

6. [10 *marks*] Find x' for $x = x(t)$ defined implicitly by

$$1 + x \ln t = te^x$$

and evaluate x' at $(t, x) = (1, 0)$.

7. [10 *marks*] The price p (in dollars) and demand x for a product are related by

$$x^2 + 2xp + 25p^2 = 74,500.$$

If the demand is decreasing at a rate of 6 per month when the demand is 150, find the rate of change of the price.

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Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209/1	All
Examination	**Date**	**Pages**
Final	December 2013	3
Instructors		**Course Examiner**
E. Duma, E. Lee, M. Padamadan		R. Raphael
R. Raphael, R. Rodriguez, F. Romanelli, C. Santana		

Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Only approved calculators allowed.

[MARKS]

[6] **1.** Find the following limits:

(a) $\lim_{x\to -2} \dfrac{2x^2+9x+10}{x+2}$ (b) $\lim_{x\to 0} \dfrac{\sqrt{49+x}-7}{x}$ (c) $\lim_{x\to\infty} \dfrac{2x^3+3x^2-9}{-\frac{1}{3}x^3+5x+7}$

[18] **2.** Find the derivative for each of the following (do not simplify):

(a) $y = 8x^3 + 7x^6 + 12$

(b) $y = \frac{2}{3}x^{-5} - 5\sqrt{x} + 5$

(c) $y = (2x^2+3)(x^3+x-2)^3$

(d) $y = \dfrac{-2x^3+x^2}{x^2+x+1}$

(e) $y = e^{\ln(7x)}$

(f) $y = (\ln(7x^3-4x))\left(e^{3x^2+15x}\right)$

(g) $y = \ln(x^3+6)^2 \cdot e^{3x^2}$

(h) Find y' and evaluate at $(1,1)$: $2y + x\ln(y) = 2x^3$

[10] 3. A company manufactures automatic transmissions for automobiles. The total weekly cost (in dollars) of producing x transmissions is given by

$$C(x) = 50,000 + 600x - 0.75.$$

(a) Find the marginal cost function.

(b) Find the marginal cost at a production level of of 200 transmissions per week and interpret the results.

(c) Find the exact cost at producing the 201st transmission.

[18] 4. For the function $f(x) = x^4 + 4x^3$ find:
[Please list the following neatly]

(a) the intervals where $f(x)$ is increasing;

(b) the intervals where $f(x)$ is decreasing;

(c) the intervals where $f(x)$ is concave up;

(d) the intervals where $f(x)$ is concave down;

(e) the local maximum;

(f) the local minimum;

(g) the inflection point(s);

(h) $\lim_{x\to+\infty} f(x)$;

(i) $\lim_{x\to-\infty} f(x)$;

(j) Using the above results, sketch the graph of $f(x)$.

[6] **5.** Find the absolute extrema of $f(x) = x^4 - 4x^3 + 5$ on the interval $[0, 4]$.

[6] **6.** Find the equation of the line tangent to the $y = -3e^{x^2} + 5$ where $x = 0$.

[6] **7.** Evaluate the following; answers must be accurate to 3 decimals:

(a) $\int_0^3 4x^2 dx$

(b) $\int_1^2 (2x + 3e^x - \frac{4}{x}) dx$

(c) $\int_0^1 xe^{-x^2} dx$

[10] **8.** Compute the antiderivatives:

(a) $\int (4t^4 - t^3 + 5t) dt$

(b) $\int \left(-\frac{3}{x} - x^{-12}\right) dx$

(c) $\int xe^{-x^2} dx$

(d) $\int (x^3 + x) e^{(x^4+2x^2)} dx$

(e) $\int (x^2 - 2)(x + 3) dx$

[10] **9.** Find the area bounded by $y = x^3$ and $y = 4x$.

[10] **10.** The Gini index of a country is $\frac{1}{6}$. Its Lorenz curve has the form $f(x) = ax + \frac{1}{2}x^2$. Find a.