Calculus III for engineers
MAT 2322 3X - Spring/Summer 2015
Practice Exam I
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1. Find and classify the critical points of the function $f(x, y) = x^4 + y^4 - 4xy + 1$.

Answer: Critical points and classification: (0,0) saddle point; (1,1) local minimum, (-1,-1) local minimum.
(See example 3 p. 947 for more details.)

2. Find the absolute maximum and minimum values of the function $f(x, y) = 2x^3 + y^4$ on the unit disk.

Answer: Absolute minimum at (-1,0) with value -2 and absolute maximum at (1,0) with value 2.
(See exercise 35 section 14.7 for more details).

3. Use the Lagrange Multiplier to find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the unit circle (or under the constraint $x^2 + y^2 = 1$).

Answer: Maximum at $(0, \pm 1)$; Minimum at $(\pm 1, 0)$.
(See example 2 section 14.8 for more details.).

3. For $f(x, y) = xe^{xy}$ and R the rectangle $[1, 2] \times [1, 3]$, what is the value of the double integral of f over R, $\iint_R f\, dA$?

Answer: $\mathrm{e}^1 - (1/3)\,\mathrm{e}^3 - \mathrm{e}^2 + (1/3)\,\mathrm{e}^6$. Note that the integration is easier if you integrate first with respect to y.

4. Let R be the region in the xy-plane bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$. Sketch the region R then evaluate the following double integral $\iint_R (x + 2y)\, dA$.

Answer: $\frac{32}{15}$.
(See example 1 section 15.3 for more details).

5. Evaluate the following double integral

$$\int_0^1 \int_y^1 x^2 \sin(xy)\, dx\, dy$$

Answer: see Question 9, Assignment 1.

6. Evaluate the following double integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^0 (x+y)\,dx\,dy$$

Answer: 0.

7. Evaluate the following triple integral

$$\int_0^1 \int_y^1 \int_0^{x+y} x\,dz\,dx\,dy$$

Answer: $\frac{3}{8}$.

8. find the mass of the solid given by the region

$$E = \{(x, y, z) |\, 0 \leq x \leq 1,\, 0 \leq y \leq 1 - x,\, 0 \leq z \leq 1 - x - y\,\}$$

if the mass density is $\rho(x, y, z) = z$.

Answer: $\frac{1}{12}$.