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CONCORDIA UNIVERSITY  
Department of Mathematics & Statistics

| Course                | Number   | Sections        |
|-----------------------|--|-----------------|
| Mathematics           | 203  | All             |
| Examination           | Date   | Pages           |
| Final                 | April 2015   | 3               |
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| Special Instructions: | Only approved calculators are allowed.<br>Show all your work for full marks. |                 |

MARKS

- [11] 1. (a) Let  $f(x) = x^2 - 4$  and  $g(x) = \sqrt{4 - x}$ . Find  $g \circ f$  and  $f \circ g$  and determine the domain of each of these composite functions.  
(b) Find the range of the function  $f = e^{2x} + 2e^x$ , the inverse function  $f^{-1}$ , and the range of  $f^{-1}$ . (HINT: assume  $e^x = u$  to see how to find  $f^{-1}$ )

- [10] 2. Evaluate the limits:  
(a)  $\lim_{x \rightarrow -2} \frac{x^2 - x - 6}{4 - x^2}$       (b)  $\lim_{x \rightarrow 0} \frac{\sqrt{x+9} - 3}{x}$

Do not use l'Hôpital rule.

- [6] 3. Find all horizontal and vertical asymptotes of the function

$$f(x) = \frac{|x|\sqrt{4x^2 + 1} - 2x^2}{x^2 - 3}$$

- [15] 4. Find the derivatives of the following functions:

- (a)  $f(x) = \frac{2\sqrt{x^5} - x^{3/2}}{x^2}$   
(b)  $f(x) = \ln \frac{x^4}{x-3}$   
(c)  $f(x) = e^3 + \arctan(e^x - e^{-x})$   
(d)  $f(x) = \frac{e^x}{1 + \cos(x^2)}$   
(e)  $f(x) = (1 + x^2)^{2x}$  (use logarithmic differentiation)

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- [15] 5. (a) Verify that the point  $(2,0)$  belongs to the curve defined by the equation  $y + x\sqrt{1+y^2} + 2 = x^2$ , and find the equation of the tangent line to the curve at this point.
- (b) A particle is moving along a circle with radius  $r = 5$  m described by the equation  $x^2 + y^2 = 25$  in the  $(x, y)$  plane. At the point  $(-4, 3)$  the  $x$ -coordinate changes at the rate  $\frac{dx}{dt} = 15 \frac{\text{m}}{\text{sec}}$ . How fast is the  $y$  coordinate changing at that instant?
- (c) Use the l'Hôpital's rule to evaluate the  $\lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2 + x^3}$ .
- [6] 6. Let  $f(x) = \frac{x}{3x - 1}$ .
- (a) Find the slope  $m$  of the secant line joining the points  $(1, f(1))$  and  $(3, f(3))$ .
- (b) Find all points  $x = c$  (if any) on the interval  $[1, 3]$  such that  $f'(c) = m$ .
- [9] 7. The volume of a sphere with radius  $r$  is given by the formula  $V(r) = \frac{4\pi}{3}r^3$ .
- (a) Use the **definition of the derivative** to show that  $\frac{dV}{dr} = 4\pi r^2$ .
- (b) If  $a$  is a given fixed value for  $r$ , write the formula for the linearization of the volume function  $V(r)$  at  $a$ .
- (c) Use this linearization to calculate the thickness  $\Delta r$  (in centimeters) of a layer of paint on the surface of a spherical ball with radius  $r = 52$  cm if the total volume of paint used is  $340 \text{ cm}^3$ .
- [12] 8. (a) Find the absolute extrema of  $f(x) = x e^{-x^2}$  on the interval  $[-\frac{1}{2}, 1]$ .
- (b) Find the radius  $r$  and the height  $h$  of the a cylindrical can that is open at the top and has a volume  $1000 \text{ cm}^3$ , but has the smallest possible surface area.

- [16] 9. Given the function  $f(x) = 2x^2 - x^4$ .
- (a) Find the domain of  $f$  and check for symmetry. Find asymptotes of  $f$  (if any).
  - (b) Calculate  $f'(x)$  and use it to determine intervals where the function is increasing, intervals where it is decreasing, and the local extrema (if any).
  - (c) Calculate  $f''(x)$  and use it to determine intervals where the function is concave upward, intervals where the function is concave downward, and the inflection points (if any).
  - (d) Sketch the graph of the function  $f(x)$  using the information obtained above.

[5] **Bonus Question**

We know that a function  $f$  is differentiable on the interval  $[0,2]$  and has values  $f(0) = 0$ ,  $f(1) = 1$  and  $f(2) = -1$ . Is this information sufficient to claim, using the Mean Value theorem, that the tangent line to the graph of  $f(x)$  must be horizontal at least at one point  $x$  in the interval  $(0,2)$ ? Explain why yes or why not.

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