MAT 1320 3X   Summer 2016   JULY 14th   Prof. Catalin Rada

TEST #2

Max = 25

-SOL-

Student Number and Name: ______________________________

- Time: 80 min.
- Only basic scientific calculators are permitted: non-programmable, non-graphing, no differentiation or integration capability. Notes or books are not permitted.
- Work all problems in the space provided. Use the backs of the pages for rough work if necessary. **Do not use any other paper!**
- Problems require complete and clearly presented solutions and carry part marks if there is substantial correct work toward the solution.

$$(xy^4 + x)' = (82)' \Rightarrow$$

0.5p

1. [1 points] If $xy^4 + x = 82$ find the equation of the tangent line to the curve y at each point where $x = 1$ and $y = 3$.

$$y^4 + x \cdot 4 \cdot y^3 y' + 1 = 0 \Rightarrow y' = \frac{-1 - y^4}{x \cdot 4 \cdot y^3}$$

if $y = mx + b$ is the sought line, then $m = \frac{-1-81}{108} = -\frac{82}{108} = -\frac{41}{54}$. So: $y = -\frac{41}{54}x + b$.

0.25p

TO GET b: solve $\Rightarrow$ $3 = -\frac{41}{54} \cdot 1 + b \Rightarrow$

$b = 3 + \frac{41}{54} = \frac{203}{54}$. $\leftarrow$ 0.25p

$$\boxed{y = -\frac{41}{54}x + \frac{203}{54}}$$

2. [3 points] Use logarithmic differentiation to find the derivative of $f(x) = \dfrac{x^{2014}\cos^{-1}x}{(x^2+1)^{2009}}$.

$$\ln f(x) = 2014 \ln x + \ln(\cos^{-1} x) - 2009 \ln(x^2+1). \leftarrow 1p$$

$$\frac{f'(x)}{f(x)} = 2014 \cdot \frac{1}{x} + \frac{\frac{-1}{\sqrt{1-x^2}}}{\cos^{-1}(x)} - 2009 \cdot \frac{2x}{x^2+1}. \quad \leftarrow 1p$$

SO:

$$f'(x) = \left[\frac{x^{2014}\cos^{-1}x}{(x^2+1)^{2009}}\right] \cdot \left[2014 \cdot \frac{1}{x} + \frac{\frac{-1}{\sqrt{1-x^2}}}{\cos^{-1}(x)} - 2009 \cdot \frac{2x}{x^2+1}\right].$$

1p

3. [5 points]

Find $\int_0^{\frac{\pi}{2}} \sin^2(x)\cos^2(x)dx$. $= \int_0^{\frac{\pi}{2}} (\sin(x)\cos(x))^2 dx =$ 1p

1p $= \int_0^{\frac{\pi}{2}} \left(\frac{\sin(2x)}{2}\right)^2 dx = \frac{1}{4}\int_0^{\frac{\pi}{2}} \sin^2(2x)dx$

$= \frac{1}{4}\int_0^{\frac{\pi}{2}} \frac{1-\cos(4x)}{2} dx =$ 1p

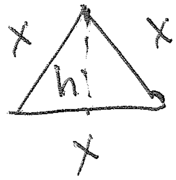
$= \frac{1}{8}\cdot\int_0^{\frac{\pi}{2}} 1-\cos(4x)dx =$

$= \frac{1}{8}\left[x - \frac{1}{4}\cdot\sin(4x)\right]\Big|_0^{\frac{\pi}{2}} = \frac{\pi}{16}$

1p 1p

$$h=\sqrt{x^2-\left(\frac{x}{2}\right)^2}=\sqrt{x^2-\frac{x^2}{4}}=\frac{\sqrt{3}}{2}x$$

4. [4 points] The side length of an equilateral triangle is increasing at a rate of 3 cm/min. How fast is the area increasing at the moment when area is 10 cm^2?



Let x = side length

$\frac{dx}{dt}=3$ 0.5p

If A = area of $\triangle$ $\Rightarrow A=\frac{x^2\sqrt{3}}{4}$ $\left(=\frac{B\cdot h}{2}\right)$ 0.5p

$\frac{dA}{dt}=\frac{\sqrt{3}}{4}\cdot 2x\,\frac{dx}{dt}$ 1p

if $A=10 \Rightarrow 10=\frac{x^2\sqrt{3}}{4} \Rightarrow x^2=\frac{40}{\sqrt{3}} \rightarrow$ 1p

$x=\sqrt{\frac{40}{\sqrt{3}}}$

In that moment the Rate is:

$$\boxed{\frac{\sqrt{3}}{4}\cdot 2\cdot\sqrt{\frac{40}{\sqrt{3}}}\cdot 3}$$ 1p

1p

$(1-x)^{\frac{1}{2}}$

5. [3 points]

Find the linear approximation of $f(x) = \sqrt{1-x}$ at $a = 0$, and use it to estimate $\sqrt{0.98}$.

$L(x) = f(0) + f'(0)(x-0) = 1 - \frac{1}{2}x$

So: $L(x) = 1 - \frac{1}{2}x$

$f(0) = \sqrt{1-0} = \sqrt{1} = 1$

$f'(x) = \frac{1}{2}(1-x)^{-\frac{1}{2}} \cdot (-1) \to f'(0) = -\frac{1}{2}$

1p

$\sqrt{0.98} = f(0.02) \approx L(0.02) = 1 - \frac{1}{2}(0.02) =$

$= 1 - 0.01 = 0.99$

1p

1p

6. [5 points] Find $\int \sqrt{1-4x^2}dx$.

$x = \frac{1}{2}\sin(\theta)$

$\frac{dx}{d\theta} = \frac{1}{2}\cos\theta$ or $dx = \frac{1}{2}\cos\theta \, d\theta$

0.5p

$\int \sqrt{1-4x^2}\,dx = \int \sqrt{1-\sin^2\theta}\ \frac{1}{2}\cos\theta \, d\theta =$

0.5p

$= \int \cos\theta \ \frac{1}{2}\cos\theta \, d\theta = \frac{1}{2}\int \cos^2\theta \, d\theta$

$= \frac{1}{2}\int \frac{\cos 2\theta + 1}{2} d\theta =$ 1p

0.5p

$= \frac{1}{4}\int (\cos 2\theta + 1) d\theta = \frac{1}{4}\left[\frac{\sin 2\theta}{2} + \theta\right] + C$

$= \frac{1}{4}\left[\sin\theta\cos\theta + \theta\right] + C$ 0.5p

$= \frac{1}{4}\left[2x\cdot\sqrt{1-4x^2} + \sin^{-1}(2x)\right] + C$

1p

Integration by parts: ☹

0.5p

0.5p

7. [4 points] Find $\int_0^1 2xe^{8x}\,dx.$ $= F(1) - F(0)$; where:

$$F(x) = \int 2xe^{8x}\,dx = \frac{e^{8x}}{8}\cdot 2x - \int \frac{e^{8x}}{8}\cdot 2\,dx$$

$$f'(x) = e^{8x} \to f(x) = \int e^{8x}\,dx = \frac{e^{8x}}{8}$$

$$g(x) = 2x \to g'(x) = 2$$

1p

$$= \frac{e^{8x}}{4}\cdot x - \frac{1}{4}\int e^{8x}\,dx =$$ ← 1p

$$= \frac{e^{8x}}{4}\cdot x - \frac{1}{4}\cdot\frac{e^{8x}}{8}.$$ ← 0.5p

Hence: our Integral becomes: $F(1) - F(0) =$

$$\frac{e^8}{4} - \frac{e^8}{32} - \left(0 - \frac{1}{32}\right) = \frac{e^8}{4} - \frac{e^8}{32} + \frac{1}{32}$$

$$= \frac{8e^8 - e^8}{32} + \frac{1}{32}$$

$$= \frac{7}{32}e^8 + \frac{1}{32}$$

0.5p

Rough work

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