an eCONCORDIA EXAMINATION

Department of Mathematics & Statistics

Course	Number	Section
MATH	209	EC
Examination	**Date**	**Pages**
Final	August 2015	3

Instructors	Course Examiners
Dr. Hal Proppe	Dr. Hal Proppe
Dr. Fred E. Szabo	Dr. Fred E. Szabo

Special Instructions

- ▷ Only approved calculators are allowed.
- ▷ Justify all your answers.

Evaluation

- ▷ This examination counts for 50% towards your final grade

MARKS

[12] **1.** (a) Find $\lim_{x\to\infty} \dfrac{7-x^2}{4x^3-6x^2+8}$.

(b) Find the value of each of the following:

i. $\lim_{x\to 5} \dfrac{x^2-25}{(x-5)}$

ii. $\lim_{x\to 4} \dfrac{x^2-3x+a}{(x-1)}$ where a is a real number.

(c) Given $\lim_{x\to 6} f(x) = -5$ and $\lim_{x\to 6} g(x) = 4$, find

i. $\lim_{x\to 6}[-3g(x)]$

ii. $\lim_{x\to 6}[f(x)]^2$

iii. $\lim_{x\to 6}[g(x)\cdot f(x)]$

[4] **2.** Using the definition of the derivative $\lim_{h\to 0} \dfrac{f(a+h)-f(a)}{h}$, find the derivative if $f(x) = 3+4x^3$.

[17] **3.** (a) If $f(x) = e - 4x^{17} - 5x^3$, find $f'(x)$.

(b) If $f(x) = (x^3 + 2x^2 - 1)(x^2 - 9)$, find $f'(x)$.

(c) If $f(x) = (7e^x + 3)/(x^2 - 4)$, find $f'(x)$.

(d) If $y = e^{(x^2-7x)}$, then $\dfrac{dy}{dx} =?$

(e) If $y = \ln(3x^3 - 5)^4$, then $\dfrac{dy}{dx} =?$

(f) If $y = \dfrac{1}{\sqrt[5]{x-2}}$, then $\dfrac{dy}{dx} =?$

(g) Find y' given $xy = e^{2y} + \pi$.

[8] **4.** Does the line tangent to the graph of $f(x) = e^x$ at $x = 1$ pass through the origin? Are there any other lines tangent to the graph of f that pass through the origin? Explain.

[5] **5.** A cube with a side of 12 centimeters is coated with ice 0.1 centimeters thick. Use differentials to estimate the volume of the ice.

[6] **6.** The total profit (in dollars) from the sale of x lawn mowers is

$$P(x) = 30x - 0.03x^2 - 750, \quad 0 \le x \le 1000$$

(a) Find the average profit per mower if 50 mowers are produced.

(b) Find the marginal average profit at a production level of 50 mowers, and interpret the results.

(c) Use the results from parts (a) and (b) to estimate the average profit per mower if 51 mowers are produced.

[5] **7.** Suppose a point is moving along the graph of $x^2 + y^2 = 34$. When the point is at $(-3, 5)$, its x coordinate is increasing at a rate of 0.6 units per second. How fast is the y coordinate changing at that moment?

[18] **8.** Compute the following:

(a) $\int x(3x^2+5)dx$

(b) $\int x^2(2x^3+7)^{18}dx$

(c) $\int \frac{1}{x^2+7}xdx$

(d) $\int e^{3-5x}dx$

(e) $\int \frac{x}{\sqrt{x+7}}dx.$

(f) $\int (x+3)^{-4}dx.$

[8] **9.** Evaluate the following integrals [give the exact answer].

(a) $\int_2^3 e^{-x^2}xdx$

(b) $\int_0^3 (t^3+5)dt$

[5] **10.** A note will pay $\$40,000$ at maturity 12 years from now. How much should you be willing to pay for the note now if money is worth 2.5%, compounded continuously?

[5] **11.** Find the area bounded by the graphs of $y = x^2 - 3x$; $y = 0$, $-2 \leq x \leq 2$.

[7] **12.** Find the interval(s) on which the graph of $f(x) = x^4 - 2x^3$ is concave upward, the interval(s) on which the graph of f is concave downward, and the inflection point(s).