

**FACULTY OF ENGINEERING AND COMPUTER SCIENCE**  
**FINAL EXAMINATION FOR APPLIED DIFFERENTIAL EQUATIONS**  
**ENGR 213 - FALL 2010**

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**Material Allowed: Calculators (non-programmable)**

**DO ALL THE PROBLEMS`**

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**Problem No. 1. (10 MARKS)** Solve the following equations using separation of variables:

(a) 
$$\frac{dy}{dx} = \frac{4 - 2x}{3y^2 - 5}$$

(b) 
$$x^2 \frac{dy}{dx} = y - xy$$

**Problem No. 2. (10 MARKS)** Solve the following equations using the exact differentials method:

(a) 
$$(2x + 3y) dx + (3x + 2y) dy = 0$$

(b) 
$$\left(1 + \ln x + \frac{y}{x}\right) dx = (1 - \ln x) dy$$

**Problem No. 3. (10 MARKS)** Solve the following Bernoulli equation

$$\frac{dy}{dx} - y = e^x y^2$$

subject to the boundary condition:  $x = 0, y = 1$ .

**Problem No. 4. (10 MARKS)** Solve the following linear differential equations using the integrating factor method:

(a) 
$$\frac{dy}{dx} - \frac{y}{x} = 1$$

$$(b) \quad x \frac{dy}{dx} + (3x + 1)y = e^{-3x}$$

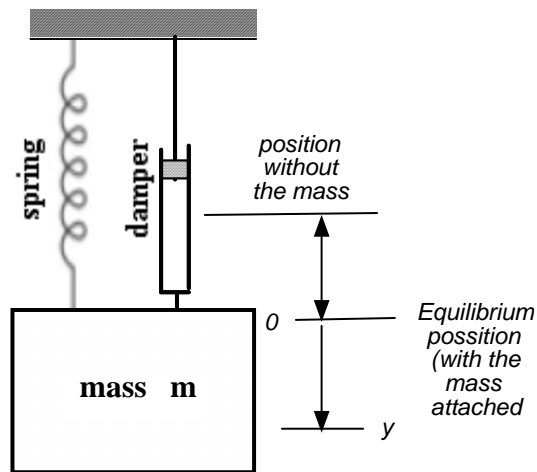
**Problem No. 5. (10 MARKS)** Give the general solutions of the following differential equations:

$$(a) \quad \frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = 0$$

$$(b) \quad \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} + 2y = 0$$

**Problem No. 6. (10 MARKS)** The equation describing the motion of the Mass - Spring-Damper system shown underneath is:

$$\frac{d^2y}{dt^2} + \frac{c}{m} \frac{dy}{dt} + \frac{k}{m} y = 0$$



where  $k$  (spring constant) =  $7/4$  N/m,  $m$  (mass) = 1 kg, and  $c$  (damper coefficient) =  $4$ Ns / m.

(a) (2 MARKS) Based on the auxiliary equation characterize the system (underdamped/critically damped/overdamped).

(b) (8 MARKS). Originally the system is stretched downwards by  $y_{in} = 0.2$  m, it is in static equilibrium, and then it is let go.

The initial conditions are:

- i.  $t = 0, y = y_{in} = 0.2$  m
- ii.  $t = 0, V_{in} = dy/dt = 0$

Find the position as function of time.

**Problem No. 7. (10 MARKS)** Solve the following differential equation by the method of undetermined coefficients:

$$y'' + 3y' + 4y = 3x + 2$$

**Problem No. 8. (10 MARKS)** Solve the following differential equation by variation of parameters:

$$y'' + 4y' + 4y = (1 + x)e^{3x}$$

**Problem No. 9. (10 MARKS).** Solve the following system of differential equations:

$$\frac{dx}{dt} = x + 2y$$

$$\frac{dy}{dt} = -x + 3y$$

**Problem No. 10. (10 MARKS).** Find the first 3 non-vanishing terms of power series solution of the equation:

$$(x^2 + 1)y'' + 2xy' = 0$$

around point  $x = 0$ .

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**USEFULL FORMULA:**  $\int \frac{dx}{ax^2 - b} = \frac{1}{2\sqrt{ab}} \ln \left| \frac{\sqrt{a}x - \sqrt{b}}{\sqrt{a}x + \sqrt{b}} \right|$